# Don Pedro Relicensing <br> W\&AR-03 and W\&AR-16 <br> River/Reservoir Temperature Models Meetings 

## Consultation Meetings Log

| Meeting Description | No. | Meeting Date |
| :--- | :---: | ---: |
| Reservoir Temperature Model | 1 | $04 / 10 / 2012$ |
| River/Reservoir Temperature Models | 2 | $10 / 26 / 2012$ |
| River/Reservoir Temperature Models | 3 | $06 / 04 / 2013$ |

## April 10, 2012 Reservoir Temperature Model Meeting No. 1 Documents:

| DATE | FROM | TO | SUBJECT |
| :--- | :--- | :--- | :--- |
| $02 / 07 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Proposed Workshops/Meetings Schedule for 2012 |
| $04 / 02 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Agenda and Advance Materials |
| $04 / 06 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Logistics |
| $07 / 26 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Draft Meeting Notes for 30-Day Review and Comment; <br> no comments received; Draft Meeting Notes are <br> considered final and are being filed with FERC here as <br> part this Draft License Application |

October 26, 2012 (postponed from September 18, 2012) River/Reservoir Temperature Models Meeting No. 2 documents:

| DATE | FROM | TO | SUBJECT |
| :--- | :--- | :--- | :--- |
| $07 / 26 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Announcement of New Date |
| $10 / 18 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Agenda and Advance Materials |
| $10 / 23 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Additional Advance Materials |
| $10 / 24 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Logistics |
| $10 / 26 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Update of Meeting Agenda Timing-running ahead of <br> schedule |
| $12 / 14 / 2012$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Draft Meeting Notes for 30-Day Review and Comment |


| DATE | FROM | TO | SUBJECT |
| :--- | :--- | :--- | :--- |
| $04 / 09 / 2013$ | Districts | FERC | Final Meeting Notes filed with FERC as part of the <br> Districts' Response to Relicensing Participants <br> Comments on the Initial Study Report |

June 4, 2013 River/Reservoir Temperature Models Meeting No. 3 Documents:

| DATE | FROM | TO | SUBJECT |
| :--- | :--- | :--- | :--- |
| $05 / 29 / 2013$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Advance Materials |
| $06 / 26 / 2013$ | Rose Staples, <br> HDR | Relicensing <br> Participants | Draft Meeting Notes for 30-Day Review and Comment |
| $07 / 19 / 2013$ | Jeffrey <br> Single, <br> CDFW | Districts | Comments on Draft Meeting Notes, as well as comments <br> on W\&AR-02 May 30, 2013 Workshop |
| $10 / 04 / 2013$ | Districts | FERC | Districts' Response to CDFW Comments |

## W\&AR-03

## Temperature Model Meeting No. 1 April 10, 2012

From:
Sent:
To:

Staples, Rose
Tuesday, February 07, 2012 8:15 PM
Alves, Jim - City of Modesto; Anderson, Craig - USFWS; Asay, Lynette - N-R; Aud, John - SCERD; Barnes, James - BLM; Barnes, Peter - SWRCB; Beuttler, John - CSPA; Blake, Martin; Bond, Jack - City of Modesto; Boucher, Allison - TRC; Boucher, Dave Allison - TRC; Bowes, Stephen - NPS; Bowman, Art - CWRMP; Brenneman, Beth - BLM; Brewer, Doug - TetraTech; Brochini, Anthony - SSMN; Brochini, Tony - NPS; Buckley, John - CSERC; Buckley, Mark; Burley, Silvia-CVMT; Burt, Charles - CalPoly; Cadagan, Jerry; Carlin, Michael - SFPUC; Catlett, Kelly - FOR; Charles, Cindy - GWWF; Cismowski, Gail - SWRCB; Costa, Jan - Chicken Ranch; Cowan, Jeffrey; Cox, Stanley Rob - TBMWI; Cranston, Peggy - BLM; Cremeen, Rebecca - CSERC; Day, Kevin - TBMI; Day, P - MF; Denean - BVR; Derwin, Maryann Moise; Devine, John; Donaldson, Milford Wayne OHP; Dowd, Maggie-SNF; Drekmeier, Peter - TRT; Edmondson, Steve - NOAA; Eicher, James - BLM; Fety, Lauren - BLM; Findley, Timothy - Hanson Bridgett; Freeman, Beau CaIPoly; Fuller, Reba - TMTC; Furman, Donn W - SFPUC; Ganteinbein, Julie - WaterPower Law Grp; Giglio, Deborah - USFWS; Gorman, Elaine - YSC; Grader, Zeke; Gutierrez, Monica - NOAA-NMFS; Hackamack, Robert; Hastreiter, James L - FERC; Hatch, Jenny - CT; Hayat, Zahra - MF; Hayden, Ann; Hellam, Anita - HH; Heyne, Tim CDFG; Holden, James ; Holm, Lisa; Horn, Jeff - BLM; Horn, Tini; Hudelson, Bill StanislausFoodProducts; Hughes, Noah; Hughes, Robert - CDFG; Hume, Noah Stillwater; Jackman, Jerry ; Jackson, Zac - USFWS; Jennings, William - CSPA; Jensen, Art - BAWSCA; Jensen, Laura - TNC; Johannis, Mary; Johnson, Brian - CalTrout; Justin; Keating, Janice; Kempton, Kathryn - NOAA-MNFS; Kinney, Teresa; Koepele, Patrick TRT; Kordella, Lesley - FERC; Lein, Joseph; Levin, Ellen - SFPUC; Lewis-Reggie-PRCl; Linkard, David - TRT /RH; Looker, Mark - LCC; Loy, Carin; Lwenya, Roselynn, BVR; Lyons, Bill - MR; Madden, Dan; Manji, Annie; Marko, Paul ; Marshall, Mike - RHH; Martin, Michael - MFFC; Martin, Ramon - USFWS; Mathiesen, Lloyd - CRRMW; McDaniel, Dan -CDWA; McDevitt, Ray - BAWSCA; McDonnell, Marty - SMRT; McLain, Jeffrey - NOAA-NMFS; Means, Julie - CDFG; Mills, John - TUD; Morningstar Pope, Rhonda - BVR; Motola, Mary - PRCl; O'Brien, Jennifer - CDFG; Orvis, Tom - SCFB; Ott, Bob; Ott, Chris; Paul, Duane - Cardno; Pavich, Steve-Cardno; Pinhey, Nick - City of Modesto; Pool, Richard; Porter, Ruth - RHH; Powell, Melissa - CRRMW; Puccini, Stephen - CDFG; Raeder, Jessie - TRT; Ramirez, Tim - SFPUC; Rea, Maria - NOAANMFS; Reed, Rhonda - NOAA-NMFS; Richardson, Kevin - USACE; Ridenour, Jim; Robbins, Royal; Romano, David O-N-R; Roos-Collins, Richard - Water-Power Law Grp for NHI; Roseman, Jesse; Rothert, Steve - AR; Sander, Max - TNC; Sandkulla, Nicole BAWSCA; Saunders, Jenan; Schutte, Allison - HB; Sears, William - SFPUC; Shakal, Sarah - Humboldt State; Shipley, Robert; Shumway, Vern - SNF; Shutes, Chris - CSPA; Sill, Todd; Slay, Ronn - CNRF/AIC; Smith, Jim - MPM; Staples, Rose; Steindorf, Dave - AW; Steiner, Dan; Stone, Vicki -TBMI; Stork, Ron - FOR; Stratton, Susan - CA SHPO; Taylor, Mary Jane - CDFG; Terpstra, Thomas; TeVelde, George A ; Thompson, Larry - NOAAMNFS; Vasquez, Sandy ; Verkuil, Colette - TRT/MF; Vierra, Chris; Villalabos, Ruben; Walters, Eric - MF; Wantuck, Rick - NOAA-NMFS; Welch, Steve - ARTA; Wesselman, Eric - TRT; Wheeler, Dan; Wheeler, Dave; Wheeler, Douglas - RHH; Wilcox, Scott Stillwater; Williamson, Harry (NPS); Willy, Alison - FWS; Wilson, Bryan - MF; Winchell, Frank - FERC; Wood, Dave - FR; Wooster, John -NOAA; Workman, Michelle - USFWS; Yoshiyama, Ron; Zipser, Wayne - SCFB
Subject:
Don Pedro Project Relicensing Water \& Aquatic Study Plans Workshop/Meeting Schedule for 2012

In accordance with FERC's Study Plan Determination and the Districts' Water \& Aquatic (W\&AR) study plans to be underway in 2012, we have developed schedule dates for the various workshops contained within the study plans. Please make note of these below:

April 2012
Apr 09 1:00 pm - 5:00 pm PT Don Pedro Project Relicensing - Hydrology Workshop (W\&AR-2) (Modesto Irrigation District Offices, Modesto \{MID\})

Apr 10 8:00 am-10:00 am PT Don Pedro Project Relicensing - Reservoir Temperature Modeling Data and Methods (MID)

Apr 10 10:15 am - 5:00 pm PT Don Pedro Project Relicensing - Salmonid Population Information Workshop (W\&AR-5) (MID)

June 2012
Jun 26 9:00 am - 4:00 pm PT Don Pedro Project Relicensing - Salmonid Population Information Workshop (W\&AR-5) (MID)

## September 2012

Sep 18 9:00 am - 4:00 pm PT Don Pedro Project Relicensing - Temperature Model Verification/Calibration Meeting (MID)

November 2012
Nov 15 9:00 am - 4:00 pm PT Don Pedro Project Relicensing - Chinook Population (W\&AR-6) and O.mykiss Population
(W\&AR-10) Modeling Workshop (MID)
In addition, in accordance with FERC's direction regarding the development and implementation of a more explicit consultation program for those studies with workshops, we are proposing to hold a meeting on March $20^{\text {th }}$ at MID (from 1:30 to 4:30 p.m.) to discuss and finalize such a Workshop Consultation Program. An initial proposal will be forwarded by March 5 to all participants.

## March 2012

Mar 20 1:30 pm-4:30 pm PT Don Pedro Project Relicensing - Workshop on Consultation Process (as per Appendix B of FERC's Study Plan Determination) (MID)

We look forward to continuing to work with all relicensing participants in 2012.

## HDR Engineering, Inc.

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| From: | Staples, Rose |
| :--- | :--- |
| Sent: | Monday, April 02, 2012 7:41 PM |
| To: | 'Alves, Jim'; 'Anderson, Craig'; 'Asay, Lynette'; 'Aud, John'; 'Barnes, James'; 'Barnes, Peter'; 'Blake, | Martin'; 'Bond, Jack'; Borovansky, Jenna; 'Boucher, Allison'; 'Bowes, Stephen'; 'Bowman, Art'; 'Brenneman, Beth'; 'Brewer, Doug'; 'Buckley, John'; 'Buckley, Mark'; 'Burley, Silvia'; 'Burt, Charles'; 'Byrd, Tim'; 'Cadagan, Jerry'; 'Carlin, Michael'; 'Charles, Cindy'; 'Cismowski, Gail'; 'Colvin, Tim'; 'Costa, Jan'; 'Cowan, Jeffrey'; 'Cox, Stanley Rob'; 'Cranston, Peggy'; 'Cremeen, Rebecca'; 'Day, Kevin'; 'Day, P'; 'Denean'; 'Derwin, Maryann Moise'; Devine, John; 'Donaldson, Milford Wayne'; 'Dowd, Maggie'; 'Drekmeier, Peter'; 'Edmondson, Steve'; 'Eicher, James'; 'Fety, Lauren'; 'Findley, Timothy'; 'Fuller, Reba'; 'Furman, Donn W'; 'Ganteinbein, Julie'; 'Giglio, Deborah'; 'Gorman, Elaine'; 'Grader, Zeke'; 'Gutierrez, Monica'; 'Hackamack, Robert'; 'Hastreiter, James'; 'Hatch, Jenny'; 'Hayat, Zahra'; 'Hayden, Ann'; 'Hellam, Anita'; 'Heyne, Tim'; 'Holley, Thomas'; 'Holm, Lisa'; 'Horn, Jeff'; 'Horn, Timi'; 'Hudelson, Bill'; 'Hughes, Noah'; 'Hughes, Robert'; 'Hume, Noah'; 'Jackman, Jerry'; 'Jackson, Zac'; 'Jennings, William'; 'Jensen, Art'; 'Jensen, Laura'; 'Johannis, Mary'; 'Johnson, Brian'; 'Justin'; 'Keating, Janice'; 'Kempton, Kathryn'; 'Kinney, Teresa'; 'Koepele, Patrick'; 'Kordella, Lesley'; 'Lein, Joseph'; 'Levin, Ellen'; 'Lewis, Reggie'; 'Linkard, David'; 'Looker, Mark'; 'Lwenya, Roselynn'; 'Lyons, Bill'; 'Madden, Dan'; 'Manji, Annie'; 'Marko, Paul'; 'Marshall, Mike'; 'Martin, Michael'; 'Martin, Ramon'; 'Mathiesen, Lloyd'; 'McDaniel, Dan'; 'McDevitt, Ray'; 'McDonnell, Marty'; 'McLain, Jeffrey'; 'Means, Julie'; 'Mills, John'; 'Morningstar Pope, Rhonda'; 'Motola, Mary'; 'O'Brien, Jennifer'; 'Orvis, Tom'; 'Ott, Bob'; 'Ott, Chris'; 'Paul, Duane'; 'Pavich, Steve'; 'Pinhey, Nick'; 'Pool, Richard'; 'Porter, Ruth'; 'Powell, Melissa'; 'Puccini, Stephen'; 'Raeder, Jessie'; 'Ramirez, Tim'; 'Rea, Maria'; 'Reed, Rhonda'; 'Richardson, Kevin'; 'Ridenour, Jim'; 'Robbins, Royal'; 'Romano, David O'; 'Roos-Collins, Richard'; 'Roseman, Jesse'; 'Rothert, Steve'; 'Sandkulla, Nicole'; 'Saunders, Jenan'; 'Schutte, Allison'; 'Sears, William'; 'Shakal, Sarah'; 'Shipley, Robert'; 'Shumway, Vern'; 'Shutes, Chris'; 'Sill, Todd'; 'Slay, Ron'; 'Smith, Jim'; Staples, Rose; 'Steindorf, Dave'; 'Steiner, Dan'; 'Stone, Vicki'; 'Stork, Ron'; 'Stratton, Susan'; 'Taylor, Mary Jane'; 'Terpstra, Thomas'; 'TeVelde, George'; 'Thompson, Larry'; 'Vasquez, Sandy'; 'Verkuil, Colette'; 'Vierra, Chris'; 'Walters, Eric'; 'Wantuck, Richard'; 'Welch, Steve'; 'Wesselman, Eric'; 'Wheeler, Dan'; 'Wheeler, Dave'; 'Wheeler, Douglas'; 'Wilcox, Scott'; 'Williamson, Harry'; 'Willy, Allison'; 'Wilson, Bryan'; 'Winchell, Frank'; 'Wooster, John'; 'Workman, Michelle'; 'Yoshiyama, Ron'; 'Zipser, Wayne'

Subject: Attachments: AGENDA-MATERIAL for W-AR-3 Reservoir Temp Modeling April 10 8:30 a.m.
ReservoirModel-April2012_ModelSteps.pdf; ReservoirModelDataTable-April2012.doc; 04-102012_DP_ReservoirTempMtg_AGENDA.doc; MIKE_321_FM_Scientific_Doc.pdf

Please find attached four documents for the Tuesday, April 10 ${ }^{\text {th }}$ (8:30 am to 10:15 am) Reservoir Temperature Modeling Data and Methods Consultation Meeting (W\&AR-3):

1. AGENDA
2. Reservoir Model Data Table
3. Reservoir Model Steps
4. Mike 21 and Mike 3 Flow Model FM Scientific Documentation

They will also be uploaded to the www.donpedro-relicensing.com website later today.
If you have any problems accessing this information, please let me know. Thank you.

# Don Pedro Relicensing 

## Reservoir Temperature Modeling Data and Methods Meeting (W\&AR-3)

Tuesday, April 10, 2012

8:30 a.m. - 10:15 a.m.

Call-In Number 866-994-6437
Conference Code 5424697994

1. Introductions and Background of Modelers
2. Overview of the Danish Hydraulic Institute and its Models
3. Overview of the MIKE3 Model
4. Model Physics and Methods
5. Model Input and Output; Data Sources
6. Schedule for Building the Model
7. Q\&A

# MIKE 21 \& MIKE 3 FLOW MODEL FM Hydrodynamic and Transport Module <br> Scientific Documentation 

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## 1 INTRODUCTION

This document presents the scientific background for the new MIKE 21 \& MIKE 3 Flow Model FM ${ }^{1}$ modelling system developed by DHI Water \& Environment. The objective is to provide the user with a detailed description of the flow and transport model equations, numerical discretization and solution methods. Also model validation is discussed in this document.

MIKE 21 \& MIKE 3 Flow Model FM is based on a flexible mesh approach and it has been developed for applications within oceanographic, coastal and estuarine environments. The modelling system may also be applied for studies of overland flooding.

The system is based on the numerical solution of the two/threedimensional incompressible Reynolds averaged Navier-Stokes equations invoking the assumptions of Boussinesq and of hydrostatic pressure. Thus, the model consists of continuity, momentum, temperature, salinity and density equations and it is closed by a turbulent closure scheme. For the 3D model the free surface is taken into account using a sigma-coordinate transformation approach.

The spatial discretization of the primitive equations is performed using a cell-centred finite volume method. The spatial domain is discretized by subdivision of the continuum into non-overlapping elements/cells. In the horizontal plane an unstructured grid is used while in the vertical domain in the 3D model a structured mesh is used. In the 2D model the elements can be triangles or quadrilateral elements. In the 3D model the elements can be prisms or bricks whose horizontal faces are triangles and quadrilateral elements, respectively.

[^0]
## 2 GOVERNING EQUATIONS

### 2.1 3D Governing Equations in Cartesian Co-ordinates

### 2.1.1 Shallow water equations

The model is based on the solution of the three-dimensional incompressible Reynolds averaged Navier-Stokes equations, subject to the assumptions of Boussinesq and of hydrostatic pressure.

The local continuity equation is written as

$$
\begin{equation*}
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=S \tag{2.1}
\end{equation*}
$$

and the two horizontal momentum equations for the x - and y component, respectively

$$
\begin{align*}
& \frac{\partial u}{\partial t}+\frac{\partial u^{2}}{\partial x}+\frac{\partial v u}{\partial y}+\frac{\partial w u}{\partial z}=f v-g \frac{\partial \eta}{\partial x}-\frac{1}{\rho_{0}} \frac{\partial p_{a}}{\partial x}- \\
& \quad \frac{g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial x} d z-\frac{1}{\rho_{0} h}\left(\frac{\partial s_{x x}}{\partial x}+\frac{\partial s_{x y}}{\partial y}\right)+F_{u}+\frac{\partial}{\partial z}\left(v_{t} \frac{\partial u}{\partial z}\right)+u_{s} S  \tag{2.2}\\
& \frac{\partial v}{\partial t}+\frac{\partial v^{2}}{\partial y}+\frac{\partial u v}{\partial x}+\frac{\partial w v}{\partial z}=-f u-g \frac{\partial \eta}{\partial y}-\frac{1}{\rho_{0}} \frac{\partial p_{a}}{\partial y}- \\
& \quad \frac{g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial y} d z-\frac{1}{\rho_{0} h}\left(\frac{\partial s_{y x}}{\partial x}+\frac{\partial s_{y y}}{\partial y}\right)+F_{v}+\frac{\partial}{\partial z}\left(v_{t} \frac{\partial v}{\partial z}\right)+v_{s} S \tag{2.3}
\end{align*}
$$

where $t$ is the time; $x, y$ and $z$ are the Cartesian co-ordinates; $\eta$ is the surface elevation; $d$ is the still water depth; $h=\eta+d$ is the total water depth; $u, v$ and $w$ are the velocity components in the $x, y$ and $z$ direction; $f=2 \Omega \sin \phi$ is the Coriolis parameter ( $\Omega$ is the angular rate of revolution and $\phi$ the geographic latitude); $g$ is the gravitational acceleration; $\rho$ is the density of water; $s_{x x}, s_{x y}, s_{y x}$ and $s_{y y}$ are components of the radiation stress tensor; $v_{t}$ is the vertical turbulent (or eddy) viscosity; $p_{a}$ is the atmospheric pressure; $\rho_{o}$ is the reference density of water. $S$ is the magnitude of the discharge due to point sources and $\left(u_{s}, v_{s}\right)$ is the velocity by which the water is discharged into the ambient water. The horizontal stress terms are described using a gradient-stress relation, which is simplified to

$$
\begin{align*}
& F_{u}=\frac{\partial}{\partial x}\left(2 A \frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(A\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)  \tag{2.4}\\
& F_{v}=\frac{\partial}{\partial x}\left(A\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(2 A \frac{\partial v}{\partial y}\right) \tag{2.5}
\end{align*}
$$

where $A$ is the horizontal eddy viscosity.
The surface and bottom boundary condition for $u, v$ and $w$ are

$$
\text { At } z=\eta \text { : }
$$

$$
\begin{equation*}
\frac{\partial \eta}{\partial t}+u \frac{\partial \eta}{\partial x}+v \frac{\partial \eta}{\partial y}-w=0, \quad\left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}\right)=\frac{1}{\rho_{0} v_{t}}\left(\tau_{s x}, \tau_{s y}\right) \tag{2.6}
\end{equation*}
$$

At $z=-d$ :

$$
\begin{equation*}
u \frac{\partial d}{\partial x}+v \frac{\partial d}{\partial y}+w=0, \quad\left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}\right)=\frac{1}{\rho_{0} v_{t}}\left(\tau_{b x}, \tau_{b y}\right) \tag{2.7}
\end{equation*}
$$

where $\left(\tau_{s x}, \tau_{s y}\right)$ and $\left(\tau_{b x}, \tau_{b y}\right)$ are the $x$ and $y$ components of the surface wind and bottom stresses.

The total water depth, $h$, can be obtained from the kinematic boundary condition at the surface, once the velocity field is known from the momentum and continuity equations. However, a more robust equation is obtained by vertical integration of the local continuity equation

$$
\begin{equation*}
\frac{\partial h}{\partial t}+\frac{\partial h \bar{u}}{\partial x}+\frac{\partial h \bar{v}}{\partial y}=h S+\widehat{P}-\widehat{E} \tag{2.8}
\end{equation*}
$$

where $\widehat{P}$ and $\widehat{E}$ are precipitation and evaporation rates, respectively, and $\bar{u}$ and $\bar{v}$ are the depth-averaged velocities

$$
\begin{equation*}
h \bar{u}=\int_{-d}^{\eta} u d z, \quad h \bar{v}=\int_{-d}^{\eta} v d z \tag{2.9}
\end{equation*}
$$

The fluid is assumed to be incompressible. Hence, the density, $\rho$, does not depend on the pressure, but only on the temperature, $T$, and the salinity, $s$, via the equation of state

$$
\begin{equation*}
\rho=\rho(T, s) \tag{2.10}
\end{equation*}
$$

Here the UNESCO equation of state is used (see UNESCO, 1981).

### 2.1.2 Transport equations for salt and temperature

The transports of temperature, $T$, and salinity, $s$, follow the general transport-diffusion equations as

$$
\begin{align*}
& \frac{\partial T}{\partial t}+\frac{\partial u T}{\partial x}+\frac{\partial v T}{\partial y}+\frac{\partial w T}{\partial z}=F_{T}+\frac{\partial}{\partial z}\left(D_{v} \frac{\partial T}{\partial z}\right)+\hat{H}+T_{s} S  \tag{2.11}\\
& \frac{\partial s}{\partial t}+\frac{\partial u s}{\partial x}+\frac{\partial v s}{\partial y}+\frac{\partial w s}{\partial z}=F_{s}+\frac{\partial}{\partial z}\left(D_{v} \frac{\partial s}{\partial z}\right)+s_{s} S \tag{2.12}
\end{align*}
$$

where $D_{v}$ is the vertical turbulent (eddy) diffusion coefficient. $\hat{H}$ is a source term due to heat exchange with the atmosphere. $T_{s}$ and $s_{s}$ are the temperature and the salinity of the source. $F$ are the horizontal diffusion terms defined by

$$
\begin{equation*}
\left(F_{T}, F_{s}\right)=\left[\frac{\partial}{\partial x}\left(D_{h} \frac{\partial}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{h} \frac{\partial}{\partial y}\right)\right](T, s) \tag{2.13}
\end{equation*}
$$

where $D_{h}$ is the horizontal diffusion coefficient. The diffusion coefficients can be related to the eddy viscosity

$$
\begin{equation*}
D_{h}=\frac{A}{\sigma_{T}} \text { and } D_{v}=\frac{v_{t}}{\sigma_{T}} \tag{2.14}
\end{equation*}
$$

where $\sigma_{T}$ is the Prandtl number. In many applications a constant Prandtl number can be used (see Rodi (1984)).

The surface and bottom boundary conditions for the temperature are

$$
\begin{align*}
& \text { At } z=\eta \text { : } \\
& D_{h} \frac{\partial T}{\partial z}=\frac{Q_{n}}{\rho_{0} c_{p}}+T_{p} \hat{P}-T_{e} \hat{E}  \tag{2.15}\\
& \text { At } z=-d: \\
& \frac{\partial T}{\partial z}=0 \tag{2.16}
\end{align*}
$$

where $Q_{n}$ is the surface net heat flux and $c_{p}=4217 \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{K}\right)$ is the specific heat of the water. A detailed description for determination of $\hat{H}$ and $Q_{n}$ is given in Section 2.7.

The surface and bottom boundary conditions for the salinity are

$$
\begin{align*}
& \text { At } z=\eta: \\
& \frac{\partial s}{\partial z}=0 \tag{2.17}
\end{align*}
$$

At $z=-d$ :

$$
\begin{equation*}
\frac{\partial s}{\partial z}=0 \tag{2.18}
\end{equation*}
$$

When heat exchange from the atmosphere is included, the evaporation is defined as

$$
\widehat{E}= \begin{cases}\frac{q_{v}}{\rho_{0} l_{v}} & q_{v}>0  \tag{2.19}\\ 0 & q_{v} \leq 0\end{cases}
$$

where $q_{v}$ is the latent heat flux and $l_{v}=2.5 \cdot 10^{6}$ is the latent heat of vaporisation of water.

### 2.1.3 Transport equation for a scalar quantity

The conservation equation for a scalar quantity is given by

$$
\begin{equation*}
\frac{\partial C}{\partial t}+\frac{\partial u C}{\partial x}+\frac{\partial v C}{\partial y}+\frac{\partial w C}{\partial z}=F_{C}+\frac{\partial}{\partial z}\left(D_{v} \frac{\partial C}{\partial z}\right)-k_{p} C+C_{s} S \tag{2.20}
\end{equation*}
$$

where $C$ is the concentration of the scalar quantity, $k_{p}$ is the linear decay rate of the scalar quantity, $C_{s}$ is the concentration of the scalar quantity at the source and $D_{v}$ is the vertical diffusion coefficient. $F_{\mathrm{F}}$ is the horizontal diffusion term defined by

$$
\begin{equation*}
F_{C}=\left[\frac{\partial}{\partial x}\left(D_{h} \frac{\partial}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{h} \frac{\partial}{\partial y}\right)\right] C \tag{2.21}
\end{equation*}
$$

where $D_{h}$ is the horizontal diffusion coefficient.

### 2.1.4 Turbulence model

The turbulence is modelled using an eddy viscosity concept. The eddy viscosity is often described separately for the vertical and the horizontal transport. Here several turbulence models can be applied: a constant viscosity, a vertically parabolic viscosity and a standard k- $\varepsilon$ model (Rodi, 1984). In many numerical simulations the small-scale turbulence can not be resolved with the chosen spatial resolution. This kind of turbulence can be approximated using sub-grid scale models.

## Vertical eddy viscosity

The eddy viscosity derived from the log-law is calculated by

$$
\begin{equation*}
v_{t}=U_{\tau} h\left(c_{1} \frac{z+d}{h}+c_{2}\left(\frac{z+d}{h}\right)^{2}\right) \tag{2.22}
\end{equation*}
$$

where $U_{\tau}=\max \left(U_{\tau \tau}, U_{\tau b}\right)$ and $c_{1}$ and $c_{2}$ are two constants. $U_{\tau s}$ and $U_{\tau b}$ are the friction velocities associated with the surface and bottom stresses, $c_{1}=0.41$ and $c_{2}=-0.41$ give the standard parabolic profile.

In applications with stratification the effects of buoyancy can be included explicitly. This is done through the introduction of a Richardson number dependent damping of the eddy viscosity coefficient, when a stable stratification occurs. The damping is a generalisation of the Munk-Anderson formulation (Munk and Anderson, 1948)

$$
\begin{equation*}
v_{t}=v_{t}^{*}(1+a R i)^{-b} \tag{2.23}
\end{equation*}
$$

where $v_{t}^{*}$ is the undamped eddy viscosity and $R i$ is the local gradient Richardson number

$$
\begin{equation*}
R i=-\frac{g}{\rho_{0}} \frac{\partial \rho}{\partial z}\left(\left(\frac{\partial u}{\partial z}\right)^{2}+\left(\frac{\partial v}{\partial z}\right)^{2}\right)^{-1} \tag{2.24}
\end{equation*}
$$

$a=10$ and $b=0.5$ are empirical constants.
In the $\mathrm{k}-\varepsilon$ model the eddy-viscosity is derived from turbulence parameters $k$ and $\varepsilon$ as

$$
\begin{equation*}
v_{t}=c_{\mu} \frac{k^{2}}{\varepsilon} \tag{2.25}
\end{equation*}
$$

where $k$ is the turbulent kinetic energy per unit mass (TKE), $\varepsilon$ is the dissipation of TKE and $c_{\mu}$ is an empirical constant.

The turbulent kinetic energy, $k$, and the dissipation of TKE, $\varepsilon$, are obtained from the following transport equations

$$
\begin{align*}
& \frac{\partial k}{\partial t}+\frac{\partial u k}{\partial x}+\frac{\partial v k}{\partial y}+\frac{\partial w k}{\partial z}=F_{k}+\frac{\partial}{\partial z}\left(\frac{v_{t}}{\sigma_{k}} \frac{\partial k}{\partial z}\right)+P+B-\varepsilon  \tag{2.26}\\
& \frac{\partial \varepsilon}{\partial t}+\frac{\partial u \varepsilon}{\partial x}+\frac{\partial v \varepsilon}{\partial y}+\frac{\partial w \varepsilon}{\partial z}= \\
& \quad F_{\varepsilon}+\frac{\partial}{\partial z}\left(\frac{v_{t}}{\sigma_{\varepsilon}} \frac{\partial \varepsilon}{\partial z}\right)+\frac{\varepsilon}{k}\left(c_{1 \varepsilon} P+c_{3 \varepsilon} B-c_{2 \varepsilon} \varepsilon\right) \tag{2.27}
\end{align*}
$$

where the shear production, $P$, and the buoyancy production, $B$, are given as

$$
\begin{align*}
& P=\frac{\tau_{x z}}{\rho_{0}} \frac{\partial u}{\partial z}+\frac{\tau_{y z}}{\rho_{0}} \frac{\partial v}{\partial z} \approx v_{t}\left(\left(\frac{\partial u}{\partial z}\right)^{2}+\left(\frac{\partial v}{\partial z}\right)^{2}\right)  \tag{2.28}\\
& B=-\frac{v_{t}}{\sigma_{t}} N^{2} \tag{2.29}
\end{align*}
$$

with the Brunt-Väisälä frequency, $N$, defined by

$$
\begin{equation*}
N^{2}=-\frac{g}{\rho_{0}} \frac{\partial \rho}{\partial z} \tag{2.30}
\end{equation*}
$$

$\sigma_{t}$ is the turbulent Prandtl number and $\sigma_{k}, \sigma_{\varepsilon}, c_{1 \varepsilon}, c_{2 \varepsilon}$ and $c_{3 \varepsilon}$ are empirical constants. $F$ are the horizontal diffusion terms defined by

$$
\begin{equation*}
\left(F_{k}, F_{\varepsilon}\right)=\left[\frac{\partial}{\partial x}\left(D_{h} \frac{\partial}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{h} \frac{\partial}{\partial y}\right)\right](k, \varepsilon) \tag{2.31}
\end{equation*}
$$

The horizontal diffusion coefficients are given by $D_{h}=A / \sigma_{k}$ and $D_{h}=A / \sigma_{\varepsilon}$, respectively.

Several carefully calibrated empirical coefficients enter the k-e turbulence model. The empirical constants are listed in (2.47) (see Rodi, 1984).

Table 2.1 Empirical constants in the $k-\varepsilon$ model.

| $c_{\mu}$ | $c_{1 \varepsilon}$ | $c_{2 \varepsilon}$ | $c_{3 \varepsilon}$ | $\sigma_{t}$ | $\sigma_{k}$ | $\sigma_{\varepsilon}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.09 | 1.44 | 1.92 | 0 | 0.9 | 1.0 | 1.3 |

At the surface the boundary conditions for the turbulent kinetic energy and its rate of dissipation depend on the wind shear, $U_{\tau 5}$

$$
\begin{align*}
& \text { At } z=\eta: \\
& k=\frac{1}{\sqrt{c_{\mu}}} U_{\tau s}^{2}  \tag{2.32}\\
& \varepsilon=\frac{U_{\tau s}^{3}}{\kappa \Delta z_{b}} \\
& \frac{\partial k}{\partial z}=0 \quad \text { for } U_{\tau \mathcal{J}}>0  \tag{2.33}\\
& \varepsilon=\frac{\left(k \sqrt{c_{\mu}}\right)^{3 / 2}}{a \kappa h} \\
& \text { for } U_{\tau \sigma}=0
\end{align*}
$$

where $\kappa=0.4$ is the von Kármán constant, $a=0.07$ is and empirical constant and $\Delta z_{s}$ is the distance from the surface where the boundary condition is imposed. At the seabed the boundary conditions are

$$
\begin{align*}
& \text { At } z=-d: \\
& k=\frac{1}{\sqrt{c_{\mu}}} U_{\tau b}^{2} \\
& \qquad \varepsilon=\frac{U_{\tau b}^{3}}{\kappa \Delta z_{b}} \tag{2.34}
\end{align*}
$$

where $\Delta z_{b}$ is the distance from the bottom where the boundary condition is imposed.

## Horizontal eddy viscosity

In many applications a constant eddy viscosity can be used for the horizontal eddy viscosity. Alternatively, Smagorinsky (1963) proposed to express sub-grid scale transports by an effective eddy viscosity related to a characteristic length scale. The subgrid scale eddy viscosity is given by

$$
\begin{equation*}
A=c_{s}^{2} l^{2} \sqrt{2 S_{i j} S_{i j}} \tag{2.35}
\end{equation*}
$$

where $c_{s}$ is a constant, $l$ is a characteristic length and the deformation rate is given by

$$
\begin{equation*}
S_{i j}=\frac{1}{2}\left(\frac{\partial u_{i}}{\partial x_{j}}+\frac{\partial u_{j}}{\partial x_{i}}\right) \quad(i, j=1,2) \tag{2.36}
\end{equation*}
$$

### 2.1.5 Governing equations in Cartesian and sigma-co-ordinates

The equations are solved using a vertical $\sigma$-transformation

$$
\begin{equation*}
\sigma=\frac{z-z_{b}}{h}, \quad x^{\prime}=x, \quad y^{\prime}=y \tag{2.37}
\end{equation*}
$$

where $\sigma$ varies between 0 at the bottom and 1 at the surface. The coordinate transformation implies relations such as

$$
\begin{align*}
& \frac{\partial}{\partial z}=\frac{1}{h} \frac{\partial}{\partial \sigma}  \tag{2.38}\\
& \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)=\left(\frac{\partial}{\partial x^{\prime}}-\frac{1}{h}\left(-\frac{\partial d}{\partial x}+\sigma \frac{\partial h}{\partial x}\right) \frac{\partial}{\partial \sigma}, \frac{\partial}{\partial y^{\prime}}-\frac{1}{h}\left(-\frac{\partial d}{\partial y}+\sigma \frac{\partial h}{\partial y}\right) \frac{\partial}{\partial \sigma}\right) \tag{2.39}
\end{align*}
$$

In this new co-ordinate system the governing equations are given as

$$
\begin{align*}
& \frac{\partial h}{\partial t}+\frac{\partial h u}{\partial x^{\prime}}+\frac{\partial h v}{\partial y^{\prime}}+\frac{\partial h \omega}{\partial \sigma}=h S  \tag{2.40}\\
& \frac{\partial h u}{\partial t}+\frac{\partial h u^{2}}{\partial x^{\prime}}+\frac{\partial h v u}{\partial y^{\prime}}+\frac{\partial h \omega u}{\partial \sigma}=f v h-g h \frac{\partial \eta}{\partial x^{\prime}}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial x^{\prime}}- \\
& \quad \frac{h g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial x} d z-\frac{1}{\rho_{0}}\left(\frac{\partial s_{x x}}{\partial x}+\frac{\partial s_{x y}}{\partial y}\right)+h F_{u}+\frac{\partial}{\partial \sigma}\left(\frac{v_{v}}{h} \frac{\partial u}{\partial \sigma}\right)+h u_{s} S  \tag{2.41}\\
& \frac{\partial h v}{\partial t}+\frac{\partial h u v}{\partial x^{\prime}}+\frac{\partial h v^{2}}{\partial y^{\prime}}+\frac{\partial h \omega v}{\partial \sigma}=-f u h-g h \frac{\partial \eta}{\partial y^{\prime}}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial y^{\prime}}- \\
& \quad \frac{h g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial y} d z-\frac{1}{\rho_{0}}\left(\frac{\partial s_{y x}}{\partial x}+\frac{\partial s_{v y}}{\partial y}\right)+h F_{v}+\frac{\partial}{\partial \sigma}\left(\frac{v_{v}}{h} \frac{\partial v}{\partial \sigma}\right)+h v_{s} S  \tag{2.42}\\
& \frac{\partial h T}{\partial t}+\frac{\partial h u T}{\partial x^{\prime}}+\frac{\partial h v T}{\partial y^{\prime}}+\frac{\partial h \omega T}{\partial \sigma}= \\
& \quad h F_{T}+\frac{\partial}{\partial \sigma}\left(\frac{D_{v}}{h} \frac{\partial T}{\partial \sigma}\right)+h \hat{H}+h T_{s} S  \tag{2.43}\\
& \frac{\partial h s}{\partial t}+\frac{\partial h u s}{\partial x^{\prime}}+\frac{\partial h v s}{\partial y^{\prime}}+\frac{\partial h \omega s}{\partial \sigma}=h F_{s}+\frac{\partial}{\partial \sigma}\left(\frac{D_{v}}{h} \frac{\partial s}{\partial \sigma}\right)+h s_{s} S \tag{2.44}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial h k}{\partial t}+\frac{\partial h u k}{\partial x^{\prime}}+\frac{\partial h v k}{\partial y^{\prime}}+\frac{\partial h \omega k}{\partial \sigma}= \\
& \quad h F_{k}+\frac{1}{h} \frac{\partial}{\partial \sigma}\left(\frac{v_{t}}{\sigma_{k}} \frac{\partial k}{\partial \sigma}\right)+h(P+B-\varepsilon)  \tag{2.45}\\
& \frac{\partial h \varepsilon}{\partial t}+\frac{\partial h u \varepsilon}{\partial x^{\prime}}+\frac{\partial h v \varepsilon}{\partial y^{\prime}}+\frac{\partial h \omega \varepsilon}{\partial \sigma}= \\
& \quad h F_{\varepsilon}+\frac{1}{h} \frac{\partial}{\partial \sigma}\left(\frac{v_{t}}{\sigma_{\varepsilon}} \frac{\partial \varepsilon}{\partial \sigma}\right)+h \frac{\varepsilon}{k}\left(c_{1 \varepsilon} P+c_{3 \varepsilon} B-c_{2 \varepsilon} \varepsilon\right)  \tag{2.46}\\
& \frac{\partial h C}{\partial t}+\frac{\partial h u C}{\partial x^{\prime}}+\frac{\partial h v C}{\partial y^{\prime}}+\frac{\partial h \omega C}{\partial \sigma}=h F_{C}+\frac{\partial}{\partial \sigma}\left(\frac{D_{v}}{h} \frac{\partial C}{\partial \sigma}\right)-h k_{p} C+h C_{s^{\prime}} \tag{2.47}
\end{align*}
$$

The modified vertical velocity is defined by

$$
\begin{equation*}
\omega=\frac{1}{h}\left[w+u \frac{\partial d}{\partial x^{\prime}}+v \frac{\partial d}{\partial y^{\prime}}-\sigma\left(\frac{\partial h}{\partial t}+u \frac{\partial h}{\partial x^{\prime}}+v \frac{\partial h}{\partial y^{\prime}}\right)\right] \tag{2.48}
\end{equation*}
$$

The modified vertical velocity is the velocity across a level of constant $\sigma$. The horizontal diffusion terms are defined as

$$
\begin{align*}
& h F_{u} \approx \frac{\partial}{\partial x}\left(2 h A \frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(h A\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)  \tag{2.49}\\
& h F_{v} \approx \frac{\partial}{\partial x}\left(h A\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left(2 h A \frac{\partial v}{\partial y}\right)  \tag{2.50}\\
& h\left(F_{T}, F_{s}, F_{k}, F_{\varepsilon}, F_{c}\right) \approx \\
& \quad\left[\frac{\partial}{\partial x}\left(h D_{h} \frac{\partial}{\partial x}\right)+\frac{\partial}{\partial y}\left(h D_{h} \frac{\partial}{\partial y}\right)\right](T, s, k, \varepsilon, C) \tag{2.51}
\end{align*}
$$

The boundary condition at the free surface and at the bottom are given as follows

At $\sigma=1$ :
$\omega=0, \quad\left(\frac{\partial u}{\partial \sigma}, \frac{\partial v}{\partial \sigma}\right)=\frac{h}{\rho_{0} v_{t}}\left(\tau_{\text {sx }}, \tau_{\text {sy }}\right)$
At $\sigma=0$ :

$$
\omega=0, \quad\left(\frac{\partial u}{\partial \sigma}, \frac{\partial v}{\partial \sigma}\right)=\frac{h}{\rho_{0} v_{t}}\left(\tau_{b x}, \tau_{b y}\right)
$$

The equation for determination of the water depth is not changed by the co-ordinate transformation. Hence, it is identical to Eq. (2.6).

### 2.2 3D Governing Equations in Spherical and Sigma Co-ordinates

In spherical co-ordinates the independent variables are the longitude, $\lambda$, and the latitude, $\phi$. The horizontal velocity field $(u, v)$ is defined by

$$
\begin{equation*}
u=R \cos \phi \frac{d \lambda}{d t} \quad v=R \frac{d \phi}{d t} \tag{2.54}
\end{equation*}
$$

where $R$ is the radius of the earth.
In this co-ordinate system the governing equations are given as (all superscripts indicating the horizontal co-ordinate in the new coordinate system are dropped in the following for notational convenience)

$$
\begin{align*}
& \frac{\partial h}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u}{\partial \lambda}+\frac{\partial h v \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega}{\partial \sigma}=h S  \tag{2.55}\\
& \frac{\partial h u}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u^{2}}{\partial \lambda}+\frac{\partial h v u \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega u}{\partial \sigma}=\left(f+\frac{u}{R} \tan \phi\right) v h- \\
& \quad \frac{1}{R \cos \phi}\left(g h \frac{\partial \eta}{\partial \lambda}+\frac{1}{\rho_{0}} \frac{\partial p_{a}}{\partial \lambda}+\frac{g}{\rho_{0}} \int_{2}^{\eta} \frac{\partial \rho}{\partial \lambda} d z+\frac{1}{\rho_{0}}\left(\frac{\partial s_{x x}}{\partial \lambda}+\cos \phi \frac{\partial s_{x y}}{\partial \phi}\right)\right)+  \tag{2.56}\\
& \quad h F_{u}+\frac{\partial}{\partial \sigma}\left(\frac{v_{v}}{h} \frac{\partial u}{\partial \sigma}\right)+h u_{s} S \\
& \frac{\partial h v}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u v}{\partial \lambda}+\frac{\partial h v^{2} \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega v}{\partial \sigma}=-\left(f+\frac{u}{R} \tan \phi\right) u h- \\
& \frac{1}{R}\left(g h \frac{\partial \eta}{\partial \phi}+\frac{1}{\rho_{0}} \frac{\partial p_{a}}{\partial \phi}+\frac{g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial \phi} d z+\frac{1}{\rho_{0}}\left(\frac{1}{\cos \phi} \frac{\partial s_{y x}}{\partial \lambda}+\frac{\partial s_{y y}}{\partial \phi}\right)\right)+  \tag{2.57}\\
& \quad h F_{v}+\frac{\partial}{\partial \sigma}\left(\frac{v_{v}}{h} \frac{\partial v}{\partial \sigma}\right)+h v_{s} S \\
& \frac{\partial h T}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u T}{\partial \lambda}+\frac{\partial h v T \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega T}{\partial \sigma}= \\
& \quad h F_{T}+\frac{\partial}{\partial \sigma}\left(\frac{D_{v}}{h} \frac{\partial T}{\partial \sigma}\right)+h \hat{H}+h T_{s} S \tag{2.58}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial h s}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u s}{\partial \lambda}+\frac{\partial h v s \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega s}{\partial \sigma}=  \tag{2.59}\\
& h F_{s}+\frac{\partial}{\partial \sigma}\left(\frac{D_{v}}{h} \frac{\partial s}{\partial \sigma}\right)+h s_{s} S \\
& \frac{\partial h k}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u k}{\partial \lambda}+\frac{\partial h v k \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega k}{\partial \sigma}= \\
& h F_{k}+\frac{1}{h} \frac{\partial}{\partial \sigma}\left(\frac{v_{t}}{\sigma_{k}} \frac{\partial k}{\partial \sigma}\right)+h(P+B-\varepsilon)  \tag{2.60}\\
& \frac{\partial h \varepsilon}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u \varepsilon}{\partial \lambda}+\frac{\partial h v \varepsilon \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega \varepsilon}{\partial \sigma}= \\
& \quad h F_{\varepsilon}+\frac{1}{h} \frac{\partial}{\partial \sigma}\left(\frac{v_{t}}{\sigma_{\varepsilon}} \frac{\partial \varepsilon}{\partial \sigma}\right)+h \frac{\varepsilon}{k}\left(c_{1 \varepsilon} P+c_{3 \varepsilon} B-c_{2 \varepsilon} \varepsilon\right)  \tag{2.61}\\
& \frac{\partial h C}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h u C}{\partial \lambda}+\frac{\partial h v C \cos \phi}{\partial \phi}\right)+\frac{\partial h \omega C}{\partial \sigma}= \\
& h F_{C}+\frac{\partial}{\partial \sigma}\left(\frac{D_{v}}{h} \frac{\partial C}{\partial \sigma}\right)-h k_{p} C+h C_{s} S \tag{2.62}
\end{align*}
$$

The modified vertical velocity in spherical co-ordinates is defined by

$$
\begin{equation*}
\omega=\frac{1}{h}\left[w+\frac{u}{R \cos \phi} \frac{\partial d}{\partial \lambda}+\frac{v}{R} \frac{\partial d}{\partial y}-\sigma\left(\frac{\partial h}{\partial t}+\frac{u}{R \cos \phi} \frac{\partial h}{\partial \lambda}+\frac{v}{R} \frac{\partial h}{\partial \phi}\right)\right] \tag{2.63}
\end{equation*}
$$

The equation determining the water depth in spherical co-ordinates is given as

$$
\begin{equation*}
\frac{\partial h}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \bar{u}}{\partial \lambda}+\frac{\partial h \bar{v} \cos \phi}{\partial \phi}\right)=h S \tag{2.64}
\end{equation*}
$$

### 2.3 2D Governing Equations in Cartesian Co-ordinates

### 2.3.1 Shallow water equations

Integration of the horizontal momentum equations and the continuity equation over depth $h=\eta+d$ the following two-dimensional shallow water equations are obtained

$$
\begin{equation*}
\frac{\partial h}{\partial t}+\frac{\partial h \bar{u}}{\partial x}+\frac{\partial h \bar{v}}{\partial y}=h S \tag{2.65}
\end{equation*}
$$

$$
\begin{gather*}
\frac{\partial h \bar{u}}{\partial t}+\frac{\partial h \bar{u}^{2}}{\partial x}+\frac{\partial h \overline{v u}}{\partial y}=f \bar{v} h-g h \frac{\partial \eta}{\partial x}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial x}- \\
\frac{g h^{2}}{2 \rho_{0}} \frac{\partial \rho}{\partial x}+\frac{\tau_{s x}}{\rho_{0}}-\frac{\tau_{b x}}{\rho_{0}}-\frac{1}{\rho_{0}}\left(\frac{\partial s_{x x}}{\partial x}+\frac{\partial s_{x y}}{\partial y}\right)+  \tag{2.66}\\
\frac{\partial}{\partial x}\left(h T_{x x}\right)+\frac{\partial}{\partial y}\left(h T_{x y}\right)+h u_{s} S \\
\frac{\partial h \bar{v}}{\partial t}+\frac{\partial h \overline{u v}}{\partial x}+\frac{\partial h \bar{v}^{2}}{\partial y}=-f \bar{u} h-g h \frac{\partial \eta}{\partial y}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial y}- \\
\frac{g h^{2}}{2 \rho_{0}} \frac{\partial \rho}{\partial y}+\frac{\tau_{s y}}{\rho_{0}}-\frac{\tau_{b y}}{\rho_{0}}-\frac{1}{\rho_{0}}\left(\frac{\partial s_{y x}}{\partial x}+\frac{\partial s_{y y}}{\partial y}\right)+  \tag{2.67}\\
\frac{\partial}{\partial x}\left(h T_{x y}\right)+\frac{\partial}{\partial y}\left(h T_{y y}\right)+h v_{s} S
\end{gather*}
$$

The overbar indicates a depth average value. For example, $\bar{u}$ and $\bar{v}$ are the depth-averaged velocities defined by

$$
\begin{equation*}
h \bar{u}=\int_{-d}^{\eta} u d z, \quad h \bar{v}=\int_{-d}^{\eta} v d z \tag{2.68}
\end{equation*}
$$

The lateral stresses $T_{i j}$ include viscous friction, turbulent friction and differential advection. They are estimated using an eddy viscosity formulation based on of the depth average velocity gradients

$$
\begin{equation*}
T_{x x}=2 A \frac{\partial \bar{u}}{\partial x}, \quad T_{x y}=A\left(\frac{\partial \bar{u}}{\partial y}+\frac{\partial \bar{v}}{\partial x}\right), \quad T_{y y}=2 A \frac{\partial \bar{v}}{\partial y} \tag{2.69}
\end{equation*}
$$

### 2.3.2 Transport equations for salt and temperature

Integrating the transport equations for salt and temperature over depth the following two-dimensional transport equations are obtained

$$
\begin{align*}
& \frac{\partial h \bar{T}}{\partial t}+\frac{\partial h \bar{u} \bar{T}}{\partial x}+\frac{\partial h \bar{v} \bar{T}}{\partial y}=h F_{T}+h \widehat{H}+h T_{s} S  \tag{2.70}\\
& \frac{\partial h \bar{s}}{\partial t}+\frac{\partial h \bar{u} \bar{s}}{\partial x}+\frac{\partial h \bar{v} \bar{s}}{\partial y}=h F_{s}+h s_{s} S \tag{2.71}
\end{align*}
$$

where $\bar{T}$ and $\bar{s}$ is the depth average temperature and salinity.

### 2.3.3 Transport equations for a scalar quantity

Integrating the transport equations for a scalar quantity over depth the following two-dimensional transport equations are obtained

$$
\begin{equation*}
\frac{\partial h \bar{C}}{\partial t}+\frac{\partial h \bar{u} \bar{C}}{\partial x}+\frac{\partial h \bar{v} \bar{C}}{\partial y}=h F_{C}-h k_{p} \bar{C}+h C_{s} S \tag{2.72}
\end{equation*}
$$

where $\bar{C}$ is the depth average scalar quantity.

### 2.4 2D Governing Equations in Spherical Co-ordinates

In spherical co-ordinates the independent variables are the longitude, $\lambda$, and the latitude, $\phi$. The horizontal velocity field ( $u, v$ ) is defined by

$$
\begin{equation*}
\bar{u}=R \cos \phi \frac{d \lambda}{d t} \quad \bar{v}=R \frac{d \phi}{d t} \tag{2.73}
\end{equation*}
$$

where $R$ is the radius of the earth.
In spherical co-ordinates the governing equation can be written

$$
\begin{align*}
& \frac{\partial h}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \bar{u}}{\partial \lambda}+\frac{\partial h \bar{v} \cos \phi}{\partial \phi}\right)=0  \tag{2.74}\\
& \frac{\partial h \bar{u}}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \bar{u}^{2}}{\partial \lambda}+\frac{\partial h \overline{v u} \cos \phi}{\partial \phi}\right)=\left(f+\frac{\bar{u}}{R} \tan \phi\right) \bar{v} h \\
& \quad-\frac{1}{R \cos \phi}\left(g h \frac{\partial \eta}{\partial \lambda}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial \lambda}+\frac{g h^{2}}{2 \rho_{0}} \frac{\partial \rho}{\partial \lambda}+\frac{1}{\rho_{0}}\left(\frac{\partial s_{x x}}{\partial \lambda}+\cos \phi \frac{\partial s_{x y}}{\partial \phi}\right)\right)+  \tag{2.75}\\
& \quad \frac{\tau_{s x}}{\rho_{0}}-\frac{\tau_{b x}}{\rho_{0}}+\frac{\partial}{\partial x}\left(h T_{x x}\right)+\frac{\partial}{\partial y}\left(h T_{x y}\right)+h u_{s} S \\
& \frac{\partial h \bar{v}}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \overline{u v}}{\partial \lambda}+\frac{\partial h \bar{v}^{2} \cos \varphi}{\partial t}\right)=-\left(f+\frac{\bar{u}}{R} \tan \phi\right) \bar{u} h \\
& -\frac{1}{R}\left(g h \frac{\partial \eta}{\partial \phi}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial \phi}+\frac{g h^{2}}{2 \rho_{0}} \frac{\partial \rho}{\partial \phi}+\frac{1}{\rho_{0}}\left(\frac{1}{\cos \phi} \frac{\partial s_{y x}}{\partial \lambda}+\frac{\partial s_{y y}}{\partial \phi}\right)\right)+  \tag{2.76}\\
& \quad \frac{\tau_{s y}}{\rho_{0}}-\frac{\tau_{b y}}{\rho_{0}}+\frac{\partial}{\partial x}\left(h T_{x y}\right)+\frac{\partial}{\partial y}\left(h T_{y y}\right)+h v_{s} S \\
& \frac{\partial h \bar{T}}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \bar{u} \bar{T}}{\partial \lambda}+\frac{\partial h \bar{v} \bar{T} \cos \phi}{\partial \phi}\right)=h F_{T}+h \bar{H}+h T_{s} S \tag{2.77}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial h \bar{s}}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \bar{u} \bar{s}}{\partial \lambda}+\frac{\partial h \bar{v} \bar{s} \cos \phi}{\partial \phi}\right)=h F_{s}+h s_{s} S  \tag{2.78}\\
& \frac{\partial h \bar{C}}{\partial t}+\frac{1}{R \cos \phi}\left(\frac{\partial h \bar{u} \bar{C}}{\partial \lambda}+\frac{\partial h \bar{v} \bar{C} \cos \phi}{\partial \phi}\right)=h F_{C}-h k_{p} \bar{C}+h C_{s} s \tag{2.79}
\end{align*}
$$

### 2.5 Bottom Stress

The bottom stress, $\vec{\tau}_{b}=\left(\tau_{b x}, \tau_{b y}\right)$, is determined by a quadratic friction law

$$
\begin{equation*}
\frac{\vec{\tau}_{b}}{\rho_{0}}=c_{f} \vec{u}_{b}\left|\vec{u}_{b}\right| \tag{2.80}
\end{equation*}
$$

where $c_{f}$ is the drag coefficient and $\vec{u}_{b}=\left(u_{b}, v_{b}\right)$ is the flow velocity above the bottom. The friction velocity associated with the bottom stress is given by

$$
\begin{equation*}
U_{\tau b}=\sqrt{c_{f}\left|u_{b}\right|^{2}} \tag{2.81}
\end{equation*}
$$

For two-dimensional calculations $\vec{u}_{b}$ is the depth-average velocity and the drag coefficient can be determined from the Chezy number, $C$, or the Manning number, $M$

$$
\begin{align*}
& c_{f}=\frac{g}{C^{2}}  \tag{2.82}\\
& c_{f}=\frac{g}{\left(M h^{1 / 6}\right)^{2}} \tag{2.83}
\end{align*}
$$

For three-dimensional calculations $\vec{u}_{b}$ is the velocity at a distance $\Delta z_{b}$ above the sea bed and the drag coefficient is determined by assuming a logarithmic profile between the seabed and a point $\Delta z_{b}$ above the seabed

$$
\begin{equation*}
c_{f}=\frac{1}{\left(\frac{1}{\kappa} \ln \left(\frac{\Delta z_{b}}{z_{0}}\right)\right)^{2}} \tag{2.84}
\end{equation*}
$$

where $\kappa=0.4$ is the von Kármán constant and $z_{0}$ is the bed roughness length scale. When the boundary surface is rough, $z_{0}$, depends on the roughness height, $k_{s}$

$$
\begin{equation*}
z_{0}=m k_{s} \tag{2.85}
\end{equation*}
$$

where m is approximately $1 / 30$.
Note, that the Manning number can be estimated from the bed roughness length using the following

$$
\begin{equation*}
M=\frac{25.4}{k_{s}^{1 / 6}} \tag{2.86}
\end{equation*}
$$

### 2.6 Wind Stress

In areas not covered by ice the surface stress, $\vec{\tau}_{s}=\left(\tau_{s x}, \tau_{s y}\right)$, is determined by the winds above the surface. The stress is given by the following empirical relation

$$
\begin{equation*}
\bar{\tau}_{s}=\rho_{a} c_{d}\left|u_{w}\right| \bar{u}_{w} \tag{2.87}
\end{equation*}
$$

where $\rho_{a}$ is the density of air, $c_{d}$ is the drag coefficient of air, and $\vec{u}_{w}=\left(u_{w}, v_{w}\right)$ is the wind speed 10 m above the sea surface. The friction velocity associated with the surface stress is given by

$$
\begin{equation*}
U_{\tau s}=\sqrt{\frac{\rho_{a} c_{f}\left|\bar{u}_{w}\right|^{2}}{\rho_{0}}} \tag{2.88}
\end{equation*}
$$

The drag coefficient can either be a constant value or depend on the wind speed. The empirical formula proposed by $\mathrm{Wu}(1980,1994)$ is used for the parameterisation of the drag coefficient.

$$
c_{f}= \begin{cases}c_{a} & w_{10}<w_{a}  \tag{2.89}\\ c_{a}+\frac{c_{b}-c_{a}}{w_{b}-w_{a}}\left(w_{10}-w_{a}\right) & w_{a} \leq w_{10}<w_{b} \\ c_{b} & w_{10} \geq w_{b}\end{cases}
$$

where $c_{a}, c_{b}, w_{a}$ and $w_{b}$ are empirical factors and $w_{10}$ is the wind velocity 10 m above the sea surface. The default values for the empirical factors are $c_{a}=1.255 \cdot 10^{-3}, c_{b}=2.425 \cdot 10^{-3}$, $w_{a}=7 \mathrm{~m} / \mathrm{s}$ and $w_{b}=25 \mathrm{~m} / \mathrm{s}$. These give generally good results for open sea applications. Field measurements of the drag coefficient collected over
lakes indicate that the drag coefficient is larger than open ocean data. For a detailed description of the drag coefficient see Geernaert and Plant (1990).

### 2.7 Ice Coverage

It is possible to take into account the effects of ice coverage on the flow field.

In areas where the sea is covered by ice the wind stress is excluded. Instead, the surface stress is caused by the ice roughness. The surface stress, $\vec{\tau}_{s}=\left(\tau_{s x}, \tau_{s y}\right)$, is determined by a quadratic friction law

$$
\begin{equation*}
\frac{\vec{\tau}_{s}}{\rho_{0}}=c_{f} \vec{u}_{s}\left|\vec{u}_{s}\right| \tag{2.90}
\end{equation*}
$$

where $c_{f}$ is the drag coefficient and $\vec{u}_{s}=\left(u_{s}, v_{s}\right)$ is the flow velocity below the surface. The friction velocity associated with the surface stress is given by

$$
\begin{equation*}
U_{\tau s}=\sqrt{c_{f}\left|u_{s}\right|^{2}} \tag{2.91}
\end{equation*}
$$

For two-dimensional calculations $\vec{u}_{s}$ is the depth-average velocity and the drag coefficient can be determined from the Manning number, $M$

$$
\begin{equation*}
c_{f}=\frac{g}{\left(M h^{1 / 6}\right)^{2}} \tag{2.92}
\end{equation*}
$$

The Manning number is estimated from the bed roughness length using the following

$$
\begin{equation*}
M=\frac{25.4}{k_{s}^{1 / 6}} \tag{2.93}
\end{equation*}
$$

For three-dimensional calculations $\vec{u}_{s}$ is the velocity at a distance $\Delta z_{s}$ below the surface and the drag coefficient is determined by assuming a logarithmic profile between the surface and a point $\Delta z_{b}$ below the surface

$$
\begin{equation*}
c_{f}=\frac{1}{\left(\frac{1}{\kappa} \ln \left(\frac{\Delta z_{s}}{z_{0}}\right)\right)^{2}} \tag{2.94}
\end{equation*}
$$

where $\kappa=0.4$ is the von Kármán constant and $z_{0}$ is the bed roughness length scale. When the boundary surface is rough, $z_{0}$, depends on the roughness height, $k_{s}$

$$
\begin{equation*}
z_{0}=m k_{s} \tag{2.95}
\end{equation*}
$$

where $m$ is approximately $1 / 30$.

### 2.8 Tidal Potential

The tidal potential is a force, generated by the variations in gravity due to the relative motion of the earth, the moon and the sun that act throughout the computational domain. The forcing is expanded in frequency space and the potential considered as the sum of a number of terms each representing different tidal constituents. The forcing is implemented as a so-called equilibrium tide, which can be seen as the elevation that theoretically would occur, provided the earth was covered with water. The forcing enters the momentum equations (e.g. (2.66) or (2.75)) as an additional term representing the gradient of the equilibrium tidal elevations, such that the elevation $\eta$ can be seen as the sum of the actual elevation and the equilibrium tidal potential.

$$
\begin{equation*}
\eta=\eta_{\text {ACTUAL }}+\eta_{T} \tag{2.96}
\end{equation*}
$$

The equilibrium tidal potential $\eta_{T}$ is given as

$$
\begin{equation*}
\eta_{T}=\sum_{i} e_{i} H_{i} f_{i} L_{i} \cos \left(2 \pi \frac{t}{T_{i}}+b_{i}+i_{0} x\right) \tag{2.97}
\end{equation*}
$$

where $\eta_{T}$ is the equilibrium tidal potential, $i$ refers to constituent number (note that the constituents here are numbered sequentially), $e_{i}$ is a correction for earth tides based on Love numbers, $H_{i}$ is the amplitude, $f_{i}$ is a nodal factor, $L_{i}$ is given below, $t$ is time, $T_{i}$ is the period of the constituent, $b_{i}$ is the phase and $x$ is the longitude of the actual position.

The phase $b$ is based on the motion of the moon and the sun relative to the earth and can be given by

$$
\begin{equation*}
b_{i}=\left(i_{1}-i_{0}\right) s+\left(i_{2}+i_{0}\right) h+i_{3} p+i_{4} N+i_{5} p_{s}+u_{i} \sin (N) \tag{2.98}
\end{equation*}
$$

where $i_{0}$ is the species, $i_{1}$ to $i_{5}$ are Doodson numbers, $u$ is a nodal modulation factor (see Table 2.3) and the astronomical arguments $s, h$, $p, N$ and $p_{s}$ are given in Table 2.2.

Table 2.2 Astronomical arguments (Pugh, 1987)

| Mean longitude of the moon | s | $277.02+481267.89 \mathrm{~T}+0.0011 \mathrm{~T}^{2}$ |
| :--- | :--- | :--- |
| Mean longitude of the sun | h | $280.19+36000.77 \mathrm{~T}+0.0003 \mathrm{~T}^{2}$ |
| Longitude of lunar perigee | p | $334.39+4069.04 \mathrm{~T}+0.0103 \mathrm{~T}^{2}$ |
| Longitude of lunar ascending node | N | $259.16+1934.14 \mathrm{~T}+0.0021 \mathrm{~T}^{2}$ |
| Longitude of perihelion | $\mathrm{p}_{\mathrm{s}}$ | $281.22+1.72 \mathrm{~T}+0.0005 \mathrm{~T}^{2}$ |

In Table 2.2 the time, T, is in Julian century from January 11900 UTC, thus $\mathrm{T}=(365(y-1900)+(d-1)+i) / 36525$ and $i=\operatorname{int}(y-$ 1901)/4), $y$ is year and $d$ is day number
$L$ depends on species number $i_{0}$ and latitude $y$ as

$$
\begin{array}{ll}
i_{0}=0 & L=3 \sin ^{2}(y)-1 \\
i_{0}=1 & L=\sin (2 y) \\
i_{0}=2 & L=\cos ^{2}(y)
\end{array}
$$

The nodal factor $f_{i}$ represents modulations to the harmonic analysis and can for some constituents be given as shown in Table 2.3.

Table 2.3 Nodal modulation terms (Pugh, 1987)

|  | $f_{i}$ | $u_{i}$ |
| :--- | :--- | :--- |
| $\mathrm{M}_{\mathrm{m}}$ | $1.000-0.130 \cos (\mathrm{~N})$ | 0 |
| $\mathrm{M}_{\mathrm{f}}$ | $1.043+0.414 \cos (\mathrm{~N})$ | $-23.7 \sin (\mathrm{~N})$ |
| $\mathrm{Q}_{1}, \mathrm{O}_{1}$ | $1.009+0.187 \cos (\mathrm{~N})$ | $10.8 \sin (\mathrm{~N})$ |
| $\mathrm{K}_{1}$ | $1.006+0.115 \cos (\mathrm{~N})$ | $-8.9 \sin (\mathrm{~N})$ |
| $2 \mathrm{~N}_{2}, \mu_{2}, \mathrm{v}_{2}, \mathrm{~N}_{2}, \mathrm{M}_{2}$ | $1.000+0.037 \cos (\mathrm{~N})$ | $-2.1 \sin (\mathrm{~N})$ |
| $\mathrm{K}_{2}$ | $1.024+0.286 \cos (\mathrm{~N})$ | $-17.7 \sin (\mathrm{~N})$ |

### 2.9 Wave Radiation

The second order stresses due to breaking of short period waves can be included in the simulation. The radiation stresses act as driving forces for the mean flow and can be used to calculate wave induced flow. For 3D simulations a simple approach is used. Here a uniform variation is used for the vertical variation in radiation stress.

### 2.10 Heat Exchange

The heat exchange with the atmosphere is calculated on basis of the four physical processes

- Latent heat flux (or the heat loss due to vaporisation)
- Sensible heat flux (or the heat flux due to convection)
- Net short wave radiation
- Net long wave radiation

Latent and sensible heat fluxes and long-wave radiation are assumed to occur at the surface. The absorption profile for the short-wave flux is approximated using Beer's law. The attenuation of the light intensity is described through the modified Beer's law as

$$
\begin{equation*}
I(d)=(1-\beta) I_{0} e^{-\lambda d} \tag{2.99}
\end{equation*}
$$

where $I(d)$ is the intensity at depth $d$ below the surface; $I_{0}$ is the intensity just below the water surface; $\beta$ is a quantity that takes into account that a fraction of light energy (the infrared) is absorbed near the surface; $\lambda$ is the light extinction coefficient. Typical values for $\beta$ and $\lambda$ are $0.2-0.6$ and $0.5-1.4 \mathrm{~m}^{0}$, respectively. $\beta$ and $\lambda$ are userspecified constants. The default values are $\beta=0.3$ and $\lambda=1.0 \mathrm{~m}^{-1}$. The fraction of the light energy that is absorbed near the surface is $\beta I_{0}$. The net short-wave radiation, $q_{s r, \text { net }}$, is attenuated as described by the modified Beer's law. Hence the surface net heat flux is given by

$$
\begin{equation*}
Q_{n}=q_{v}+q_{c}+\beta q_{s r, n e t}+q_{l r, n e t} \tag{2.100}
\end{equation*}
$$

For three-dimensional calculations the source term $\hat{H}$ is given by

$$
\begin{equation*}
\hat{H}=\frac{\partial}{\partial z}\left(\frac{q_{s r, n e t}(1-\beta) e^{-\lambda(\eta-z)}}{\rho_{0} c_{p}}\right)=\frac{q_{s r, n e t}(1-\beta) \frac{e^{-\lambda(\eta-z)}}{\lambda}}{\rho_{0} c_{p}} \tag{2.101}
\end{equation*}
$$

For two-dimensional calculations the source term $\hat{H}$ is given by

$$
\begin{equation*}
\hat{H}=\frac{q_{v}+q_{c}+q_{s r, \text { net }}+q_{l r, \text { net }}}{\rho_{0} c_{p}} \tag{2.102}
\end{equation*}
$$

The calculation of the latent heat flux, sensible heat flux, net short wave radiation, and net long wave radiation as described in the following sections.

In areas covered by ice the heat exchange is excluded.

### 2.10.1 Vaporisation

Dalton's law yields the following relationship for the vaporative heat loss (or latent flux), see Sahlberg, 1984

$$
\begin{equation*}
q_{v}=L C_{e}\left(a_{1}+b_{1} W_{2 m}\right)\left(Q_{\text {water }}-Q_{\text {air }}\right) \tag{2.103}
\end{equation*}
$$

where $L=2.5 \cdot 10^{6} \mathrm{~J} / \mathrm{kg}$ is the latent heat vaporisation (in the literature $L=2.5 \cdot 10^{6}-2300 T_{\text {water }}$ is commonly used); $C_{e}=1.32 \cdot 10^{-3}$ is the moisture transfer coefficient (or Dalton number); $W_{2 m}$ is the wind speed 2 m above the sea surface; $Q_{\text {water }}$ is the water vapour density close to the surface; $Q_{\text {air }}$ is the water vapour density in the atmosphere; $a_{1}$ and $b_{1}$ are user specified constants. The default values are $a_{1}=0.5$ and $b_{1}=0.9$.

Measurements of $Q_{\text {water }}$ and $Q_{\text {air }}$ are not directly available but the vapour density can be related to the vapour pressure as

$$
\begin{equation*}
Q_{i}=\frac{0.2167}{T_{i}+T_{k}} e_{i} \tag{2.104}
\end{equation*}
$$

in which subscript $i$ refers to both water and air. The vapour pressure close to the sea, $e_{\text {water }}$, can be expressed in terms of the water temperature assuming that the air close to the surface is saturated and has the same temperature as the water

$$
\begin{equation*}
e_{\text {water }}=6.11 e^{K}\left(\frac{1}{T_{k}}-\frac{1}{T_{\text {water }}+T_{k}}\right) \tag{2.105}
\end{equation*}
$$

where $K=5418^{\circ} \mathrm{K}$ and $T_{K}=273.15^{\circ} \mathrm{K}$ is the temperature at 0 C . Similarly the vapour pressure of the air, $e_{\text {air }}$, can be expressed in terms of the air temperature and the relative humidity, R

$$
\begin{equation*}
e_{\text {air }}=R \cdot 6.11 e^{K}\left(\frac{1}{T_{k}}-\frac{1}{T_{\text {air }}+T_{k}}\right) \tag{2.106}
\end{equation*}
$$

Replacing $Q_{\text {water }}$ and $Q_{\text {air }}$ with these expressions the latent heat can be written as

$$
\begin{align*}
q_{v} & =-P_{v}\left(a_{1}+b_{1} W_{2 m}\right) . \\
& \left(\frac{\exp \left(K\left(\frac{1}{T_{k}}-\frac{1}{T_{\text {water }}+T_{k}}\right)\right)}{T_{\text {water }}+T_{k}}-\frac{R \cdot \exp \left(K\left(\frac{1}{T_{k}}-\frac{1}{T_{\text {air }}+T_{k}}\right)\right)}{T_{\text {air }}+T_{k}}\right. \tag{2.107}
\end{align*}
$$

where all constants have been included in a new latent constant $P_{v}=4370 \mathrm{~J} \cdot{ }^{\circ} \mathrm{K} / \mathrm{m}^{3}$. During cooling of the surface the latent heat loss has a major effect with typical values up to $100 \mathrm{~W} / \mathrm{m}^{5}$.

### 2.10.2 Convection

The sensible heat flux, $q_{c}\left(W / \mathrm{m}^{2}\right)$, (or the heat flux due to convection) depends on the type of boundary layer between the sea surface and the atmosphere. Generally this boundary layer is turbulent implying the following relationship

$$
q_{c}=\left\{\begin{array}{lc}
\rho_{\text {air }} c_{\text {air }} c_{\text {heating }} W_{10}\left(T_{\text {air }}-T_{\text {water }}\right) & T_{\text {air }} \geq T  \tag{2.108}\\
\rho_{\text {air }} c_{\text {air }} c_{\text {cooling }} W_{10}\left(T_{\text {air }}-T_{\text {water }}\right) & T_{\text {air }}<T
\end{array}\right.
$$

where $\rho_{\text {air }}$ is the air density $1.225 \mathrm{~kg} / \mathrm{m}^{6} \# \mathrm{c}_{\text {air }}=1007 \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{K}\right)$ is the specific heat of air; $c_{\text {heating }}=0.0011$ and $c_{\text {cooling }}=0.0011$, respectively, is the sensible transfer coefficient (or Stanton number) for heating and cooling (see Kantha and Clayson, 2000); $W_{10}$ is the wind speed 10 m above the sea surface; $T_{\text {water }}$ is the temperature at the sea surface; $T_{\text {air }}$ is the temperature of the air.

The convective heat flux typically varies between 0 and $100 \mathrm{~W} / \mathrm{m}^{5} 1$

### 2.10.3 Short wave radiation

Radiation from the sun consists of electromagnetic waves with wave lengths varying from 1,000 to $30,000 \AA$. Most of this is absorbed in the ozone layer, leaving only a fraction of the energy to reach the surface of the Earth. Furthermore, the spectrum changes when sunrays pass through the atmosphere. Most of the infrared and ultraviolet compound is absorbed such that the solar radiation on the Earth mainly consists of light with wave lengths between 4,000 and $9,000 \AA$. This radiation is normally termed short wave radiation. The intensity depends on the distance to the sun, declination angle and latitude,
extraterrestrial radiation and the cloudiness and amount of water vapour in the atmosphere (see Iqbal, 1983)

The eccentricity in the solar orbit, $E_{0}$, is given by

$$
\begin{align*}
E_{0}=\left(\frac{r_{0}}{r}\right)^{2}= & 1.000110+0.034221 \cos (\Gamma)+0.001280 \sin (\Gamma)  \tag{2.109}\\
& +0.000719 \cos (2 \Gamma)+0.000077 \sin (2 \Gamma)
\end{align*}
$$

where $r_{0}$ is the mean distance to the sun, $r$ is the actual distance and the day angle $\Gamma(\mathrm{rad})$ is defined by

$$
\begin{equation*}
\Gamma=\frac{2 \pi\left(d_{n}-1\right)}{365} \tag{2.110}
\end{equation*}
$$

and $d_{n}$ is the Julian day of the year.

The daily rotation of the Earth around the polar axes contributes to changes in the solar radiation. The seasonal radiation is governed by the declination angle, $\delta(\mathrm{rad})$, which can be expressed by

$$
\begin{gather*}
\delta=0.006918-0.399912 \cos (\Gamma)+0.07257 \sin (\Gamma)- \\
0.006758 \cos (2 \Gamma)+0.000907 \sin (2 \Gamma)-  \tag{2.111}\\
0.002697 \cos (3 \Gamma)+0.00148 \sin (3 \Gamma)
\end{gather*}
$$

The day length, $n_{d}$, varies with $\delta$. For a given latitude, $\phi$, (positive on the northern hemisphere) the day length is given by

$$
\begin{equation*}
n_{d}=\frac{24}{\pi} \arccos (-\tan (\phi) \tan (\delta)) \tag{2.112}
\end{equation*}
$$

and the sunrise angle, $\omega_{s r}(\mathrm{rad})$, and the sunset angle $\omega_{s s}(\mathrm{rad})$ are

$$
\begin{equation*}
\omega_{s r}=\arccos (-\tan (\phi) \tan (\delta)) \quad \text { and } \quad \omega_{s s}=\omega_{s r} \tag{2.113}
\end{equation*}
$$

The intensity of short wave radiation on the surface parallel to the surface of the Earth changes with the angle of incidence. The highest intensity is in zenith and the lowest during sunrise and sunset. Integrated over one day the extraterrestrial intensity,
$H_{0}\left(M J / \mathrm{m}^{2} /\right.$ day $)$, in short wave radiation on the surface can be derived as

$$
\begin{equation*}
H_{0}=\frac{24}{\pi} q_{s c} E_{0} \cos (\phi) \cos (\delta)\left(\sin \left(\omega_{s r}\right)-\omega_{s r} \cos \left(\omega_{s r}\right)\right) \tag{2.114}
\end{equation*}
$$

where $q_{s c}=4.9212\left(M J / \mathrm{m}^{2} / h\right)$ is the solar constant.
For determination of daily radiation under cloudy skies, $H$ ( $M J / \mathrm{m}^{2} /$ day ), the following relation is used

$$
\begin{equation*}
\frac{H}{H_{0}}=a_{2}+b_{2} \frac{n}{n_{d}} \tag{2.115}
\end{equation*}
$$

in which $n$ is the number of sunshine hours and $n_{d}$ is the maximum number of sunshine hours. $a_{2}$ and $b_{2}$ are user specified constants. The default values are $a_{2}=0.295$ and $b_{2}=0.371$. The user-specified clearness coefficient corresponds to $n / n_{d}$. Thus the average hourly short wave radiation, $q_{s}\left(M J / m^{2} / h\right)$, can be expressed as

$$
\begin{equation*}
q_{s}=\left(\frac{H}{H_{0}}\right) q_{0}\left(a_{3}+b_{3} \cos \left(\omega_{i}\right)\right) \tag{2.116}
\end{equation*}
$$

where

$$
\begin{align*}
& a_{3}=0.4090+0.5016 \sin \left(\omega_{s r}-\frac{\pi}{3}\right)  \tag{2.117}\\
& b_{3}=0.6609+0.4767 \sin \left(\omega_{s r}-\frac{\pi}{3}\right) \tag{2.118}
\end{align*}
$$

The extraterrestrial intensity, $q_{0}\left(M J / m^{2} / h\right)$ and the hour angle $\omega_{i}$ is given by

$$
\begin{align*}
& q_{0}=q_{s c} E_{0}\left(\sin (\phi) \sin (\delta)+\frac{24}{\pi} \cos (\phi) \cos (\delta) \cos \left(\omega_{i}\right)\right)  \tag{2.119}\\
& \omega_{i}=\frac{\pi}{12}\left(12+\Delta t_{\text {displacement }}+\frac{4}{60}\left(L_{S}-L_{E}\right)-\frac{E_{t}}{60}-t_{\text {local }}\right) \tag{2.120}
\end{align*}
$$

$\Delta t_{\text {displacement }}$ is the displacement hours due to summer time and the time meridian $L_{S}$ is the standard longitude for the time zone.
$\Delta t_{\text {displacement }}$ and $L_{s}$ are user specified constants. The default values are $\Delta t_{\text {displacement }}=0(h)$ and $L_{S}=0(\mathrm{deg}) \cdot L_{E}$ is the local longitude in degrees. $E_{t}(s)$ is the discrepancy in time due to solar orbit and is varying during the year. It is given by

$$
\begin{equation*}
E_{t}=\binom{0.000075+0.001868 \cos (\Gamma)-0.032077 \sin (\Gamma)}{-0.014615 \cos (2 \Gamma)-0.04089 \sin (2 \Gamma)} \cdot 229.18 \tag{2.121}
\end{equation*}
$$

Finally, $t_{\text {local }}$ is the local time in hours.
Solar radiation that impinges on the sea surface does not all penetrate the water surface. Parts are reflected back and are lost unless they are backscattered from the surrounding atmosphere. This reflection of solar energy is termed the albedo. The amount of energy, which is lost due to albedo, depends on the angle of incidence and angle of refraction. For a smooth sea the reflection can be expressed as

$$
\begin{equation*}
\alpha=\frac{1}{2}\left(\frac{\sin ^{2}(i-r)}{\sin ^{2}(i+r)}+\frac{\tan ^{2}(i-r)}{\tan ^{2}(i+r)}\right) \tag{2.122}
\end{equation*}
$$

where $i$ is the angle of incidence, $r$ the refraction angle and $\alpha$ the reflection coefficient, which typically varies from 5 to $40 \%$. $\alpha$ can be approximated using

$$
\alpha= \begin{cases}\frac{\text { altitude }}{5} 0.48 & \text { altitude }<5  \tag{2.123}\\ \frac{30-\text { altitude }}{25}(0.48-0.05) & 5 \leq \text { altitude } \leq 30 \\ 0.05 & \text { altitude }>30\end{cases}
$$

where the altitude in degrees is given by

$$
\begin{equation*}
\text { altitude }=90-\left(\frac{180}{\pi} \arccos \left(\sin (\delta) \sin (\phi)+\cos (\delta) \cos (\phi) \cos \left(\omega_{i}\right)\right)\right) \tag{2.124}
\end{equation*}
$$

Thus the net short wave radiation, $q_{s, n e t}\left(W / m^{2}\right)$, can eventually be expressed as

$$
\begin{equation*}
q_{s r, n e t}=(1-\alpha) q_{\mathrm{s}} \frac{10^{6}}{3600} \tag{2.125}
\end{equation*}
$$

### 2.10.4 Long wave radiation

A body or a surface emits electromagnetic energy at all wavelengths of the spectrum. The long wave radiation consists of waves with wavelengths between 9,000 and $25,000 \AA$. The radiation in this interval is termed infrared radiation and is emitted from the
atmosphere and the sea surface. The long wave emittance from the surface to the atmosphere minus the long wave radiation from the atmosphere to the sea surface is called the net long wave radiation and is dependent on the cloudiness, the air temperature, the vapour pressure in the air and the relative humidity. The net outgoing long wave radiation, $q_{l r, n e t}\left(W / m^{2}\right)$, is given by Brunt's equation (See Lind and Falkenmark, 1972)

$$
\begin{equation*}
q_{l r, n e t}=-\sigma_{s b}\left(T_{a i r}+T_{K}\right)^{4}\left(a-b \sqrt{e_{d}}\right)\left(c+d \frac{n}{n_{d}}\right) \tag{2.126}
\end{equation*}
$$

where $e_{d}$ is the vapour pressure at dew point temperature measured in $m b ; n$ is the number of sunshine hours, $n_{d}$ is the maximum number of sunshine hours; $\sigma_{s b}=5.6697 \cdot 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{K}^{4}\right)$ is Stefan Boltzman's constant; $T_{\text {air }}\left({ }^{\circ} \mathrm{C}\right)$ is the air temperature. The coefficients $a, b, c$ and $d$ are given as

$$
\begin{equation*}
a=0.56 ; b=0.077 m b^{-1 / 2} ; c=0.10 ; d=.90 \tag{2.127}
\end{equation*}
$$

The vapour pressure is determined as

$$
\begin{equation*}
e_{d}=10 \cdot R e_{\text {saturated }} \tag{2.128}
\end{equation*}
$$

where $R$ is the relative humidity and the saturated vapour pressure, $e_{\text {saturated }}(k P a)$, with $100 \%$ relative humidity in the interval from -51 to $52^{\circ} \mathrm{C}$ can be estimated by

$$
\begin{align*}
& e_{\text {saturated }}=3.38639 \\
& \quad\left(\left(7.38 \cdot 10^{-3} \cdot T_{\text {air }}+0.8072\right)^{8}-1.9 \cdot 10^{-5}\left|1.8 \cdot T_{\text {air }}+48\right|+1.316 \cdot 10^{-3}\right) \tag{2.129}
\end{align*}
$$

## 3 NUMERICAL SOLUTION

### 3.1 Spatial Discretization

The discretization in solution domain is performed using a finite volume method. The spatial domain is discretized by subdivision of the continuum into non-overlapping cells/elements.

In the two-dimensional case the elements can be arbitrarily shaped polygons, however, here only triangles and quadrilateral elements are considered.

In the three-dimensional case a layered mesh is used: in the horizontal domain an unstructured mesh is used while in the vertical domain a structured mesh is used (see Figure 3.1gure 3.1). The vertical mesh is based on either sigma coordinates or combined sigma/z-level coordinates. For the hybrid sigma/z-level mesh sigma coordinates are used from the free surface to a specified depth and z-level coordinates are used below. The different types of vertical mesh are illustrated in Figure 3.2. The elements in the sigma domain and the z-level domain can be prisms with either a 3-sided or 4-sided polygonal base. Hence, the horizontal faces are either triangles or quadrilateral element. The elements are perfectly vertical and all layers have identical topology.


Figure 3.1 Principle of meshing for the three-dimensional case


Figure 3.2 Illustrations of the different vertical grids. Upper: sigma mesh, Lower: combined sigma/z-level mesh with simple bathymetry adjustment. The red line shows the interface between the z-level domain and the sigmalevel domain

The most important advantage using sigma coordinates is their ability to accurately represent the bathymetry and provide consistent resolution near the bed. However, sigma coordinates can suffer from significant errors in the horizontal pressure gradients, advection and mixing terms in areas with sharp topographic changes (steep slopes). These errors can give rise to unrealistic flows.

The use of z-level coordinates allows a simple calculation of the horizontal pressure gradients, advection and mixing terms, but the disadvantages are their inaccuracy in representing the bathymetry and that the stair-step representation of the bathymetry can result in unrealistic flow velocities near the bottom.

### 3.1.1 Vertical Mesh

For the vertical discretization both a standard sigma mesh and a combined sigma/z-level mesh can be used. For the hybrid sigma/zlevel mesh sigma coordinates are used from the free surface to a specified depth, $z_{\sigma}$, and z-level coordinates are used below. At least one sigma layer is needed to allow changes in the surface elevation.

## Sigma

In the sigma domain a constant number of layers, $N_{\sigma}$, are used and each sigma layer is a fixed fraction of the total depth of the sigma layer, $h_{\sigma}$, where $h_{\sigma}$ A $\eta-q$ e $\mathcal{Z a z}_{b} \propto_{\sigma}$. The discretization in the sigma domain is given by a number of discrete $\sigma$-levels $\left\{\sigma_{i} \mathrm{~A} 50, N_{\sigma} /\right.$ $5-\}$ Here $\sigma$ varies from $\sigma_{5}$ A 4 at the bottom interface of the lowest sigma layer to $\sigma_{N_{\sigma} / 5} \mathrm{~A} 5$ at the free surface.

Variable sigma coordinates can be obtained using a discrete formulation of the general vertical coordinate (s-coordinate) system proposed by Song and Haidvogel (1994). First an equidistant discretization in a s-coordinate system ( $-1 \leq s \leq 0$ ) is defined

$$
\begin{equation*}
s_{i} \mathrm{~A}-\frac{N_{\sigma} / 5-i}{N_{\sigma}} \quad i \text { A } 50, N_{\sigma} / 5- \tag{3.1}
\end{equation*}
$$

The discrete sigma coordinates can then be determined by

$$
\begin{equation*}
\sigma_{i}=1+\sigma_{c} s_{i}+\left(1-\sigma_{c}\right) c\left(s_{i}\right) \quad i=1,\left(N_{\sigma}+1\right) \tag{3.2}
\end{equation*}
$$

where

$$
\begin{equation*}
c(s) A(5-b) \frac{\operatorname{wrrl}(\theta s)}{\operatorname{wrrl}(\theta)} / b \frac{\operatorname{xerl}\left(\theta\left(s / \frac{5}{6}\right)\right)-\operatorname{xerl} \frac{\theta}{6}-\frac{\theta}{6}}{6 \operatorname{xerl} \frac{\theta}{6}-} \tag{3.3}
\end{equation*}
$$

Here $\sigma_{c}$ is a weighting factor between the equidistant distribution and the stretch distribution, $\theta$ is the surface control parameter and $b$ is the bottom control parameter. The range for the weighting factor is $0<\sigma_{c} \leq l$ where the value 1 corresponds to equidistant distribution and 0 corresponds to stretched distribution. A small value of $\sigma_{c}$ can result in linear instability. The range of the surface control parameter is $0<\theta \leq 20$ and the range of the bottom control parameter is $0 \leq b \leq 1$. If $\theta \ll 1$ and $b=0$ an equidistant vertical resolution is obtained. By increasing the value of the $\theta$, the highest resolution is achieved near the surface. If $\theta>0$ and $b=1$ a high resolution is obtained both near the surface and near the bottom.

Examples of a mesh using variable vertical discretization are shown in Figure 3.3 and Figure 3.4.


Figure 3.3 Example of vertical distribution using layer thickness distribution. Number of layers: 10, thickness of layers 1 to 10: .025, 0.075, 0.1, $0.01,0.02,0.02,0.1,0.1,0.075,0.025$


Figure 3.4 Example of vertical distribution using variable distribution. Number of layers: $10, \sigma_{c}=0.1, \theta=5, b=1$

## Combined sigma/z-level

In the z-level domain the discretization is given by a number of discrete z-levels $\left\{z_{i}\right.$ A $50, N_{z} / 5$-\} in the z-level domain. $z_{1}$ is the minimum z-level and $z_{N_{z} / 5}$ is the maximum z-level, which is equal to the sigma depth, $z_{\sigma}$. The corresponding layer thickness is given by

$$
\begin{equation*}
\Delta z_{i} \text { A } z_{i / 5}-z_{i} \quad i \text { A } 50 N_{z} \tag{3.4}
\end{equation*}
$$

The discretization is illustrated in Figure 3.5 and Figure 3.6.
Using standard z-level discretization the bottom depth is rounded to the nearest $z$-level. Hence, for a cell in the horizontal mesh with the cell-averaged depth, $z_{b}$, the cells in the corresponding column in the z domain are included if the following criteria is satisfied

$$
\begin{equation*}
\left(\sim_{n / 5}-\sim_{n}\right) 36 \geq z_{b} \quad i \text { A } 50 N_{z} \tag{3.5}
\end{equation*}
$$

The cell-averaged depth, $z_{b}$, is calculated as the mean value of the depth at the vortices of each cell. For the standard z-level discretization the minimum depth is given by $z_{1}$. Too take into account the correct depth for the case where the bottom depth is below the minimum z-level ( $z_{5} \mathrm{~B} \quad z_{b}$ ) a bottom fitted approach is used. Here, a correction factor, $f_{1}$, for the layer thickness in the bottom cell is introduced. The correction factor is used in the calculation of the volume and face integrals. The correction factor for the bottom cell is calculated by

$$
\begin{equation*}
f_{5} \mathrm{~A} \frac{, z_{6}-z_{b}-}{\Delta z_{5}} \tag{3.6}
\end{equation*}
$$

The corrected layer thickness is given by $\Delta z_{5}^{*} \mathrm{~A} f_{5} \Delta z_{5}$. The simple bathymetry adjustment approach is illustrated in Figure 3.5.

For a more accurate representation of the bottom depth an advanced bathymetry adjustment approach can be used. For a cell in the horizontal mesh with the cell-averaged depth $\$ \neq z$, the cells in the corresponding column in the z -domain are included if the following criteria is satisfied

$$
\begin{equation*}
\sim_{\mathrm{r} / 5} \mathrm{~B} z_{b} \quad i \text { A } 50 N_{z} \tag{3.7}
\end{equation*}
$$

A correction factor, $f_{i}$, is introduced for the layer thickness

$$
\begin{equation*}
f_{i} \mathrm{~A} \max \left(\frac{z_{i / 5}-z_{b}-z_{\min }}{\Delta z_{i}} \frac{\mathrm{~B} z_{b}}{\Delta z_{i}}\right) \quad z_{i} @ z_{b} @ z_{i / 5} \text { or } z_{5} \tag{3.8}
\end{equation*}
$$

$$
f_{i} \text { A } 5
$$

A minimum layer thickness, $\Delta z_{\text {min }}$, is introduced to avoid very small values of the correction factor. The correction factor is used in the calculation of the volume and face integrals. The corrected layer thicknesses are given by $\left\{\Delta z_{i}^{*}\right.$ A $f_{i} \Delta z_{i} 0 i$ A $\left.50 N_{z}\right\}$ 冬男he advanced bathymetry adjustment approach is illustrated in Figure 3.6.


Figure 3.5 Simple bathymetry adjustment approach


Figure 3.6 Advanced bathymetry adjustment approach

### 3.1.2 Shallow water equations

The integral form of the system of shallow water equations can in general form be written

$$
\begin{equation*}
\frac{\partial \boldsymbol{U}}{\partial t}+\nabla \cdot \boldsymbol{F}(\boldsymbol{U})=\boldsymbol{S}(\boldsymbol{U}) \tag{3.9}
\end{equation*}
$$

where $\boldsymbol{U}$ is the vector of conserved variables, $\boldsymbol{F}$ is the flux vector function and $\boldsymbol{S}$ is the vector of source terms.

In Cartesian co-ordinates the system of 2D shallow water equations can be written

$$
\begin{equation*}
\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial\left(\boldsymbol{F}_{x}^{I}-\boldsymbol{F}_{x}^{V}\right)}{\partial x}+\frac{\partial\left(\boldsymbol{F}_{y}^{I}-\boldsymbol{F}_{y}^{V}\right)}{\partial y}=\boldsymbol{S} \tag{3.10}
\end{equation*}
$$

where the superscripts $I$ and $V$ denote the inviscid (convective) and viscous fluxes, respectively and where

$$
\begin{align*}
& \boldsymbol{U}=\left[\begin{array}{l}
h \\
h \bar{u} \\
h \bar{v}
\end{array}\right], \\
& \boldsymbol{F}_{x}{ }^{I}=\left[\begin{array}{l}
h \bar{u} \\
h \bar{u}^{2}+\frac{1}{2} g\left(h^{2}-d^{2}\right) \\
h \overline{u v}
\end{array}\right], \quad \mathrm{F}_{x}^{V}=\left[\begin{array}{l}
0 \\
h A\left(2 \frac{\partial \bar{u}}{\partial x}\right) \\
h A\left(\frac{\partial \bar{u}}{\partial y}+\frac{\partial \bar{v}}{\partial x}\right)
\end{array}\right] \\
& \boldsymbol{F}_{y}{ }^{I}=\left[\begin{array}{l}
h \bar{v} \\
h \overline{\nu u} \\
h \bar{v}^{2}+\frac{1}{2} g\left(h^{2}-d^{2}\right)
\end{array}\right]_{a}, \quad \mathrm{~F}_{y}{ }^{V}=\left[\begin{array}{l}
0 \\
h A\left(\frac{\partial \bar{u}}{\partial y}+\frac{\partial \bar{v}}{\partial x}\right) \\
h A\left(2 \frac{\partial \bar{v}}{\partial x}\right)
\end{array}\right]  \tag{3.11}\\
& \boldsymbol{S}=\left[\begin{array}{c}
0 \\
g \eta \frac{\partial d}{\partial x}+f \bar{v} h-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial x}-\frac{g h^{2}}{2 \rho_{0}} \frac{\partial \rho}{\partial x}-\frac{1}{\rho_{0}}\left(\frac{\partial s_{x x}}{\partial x}+\frac{\partial s_{x y}}{\partial y}\right) \\
+\frac{\tau_{s x}}{\rho_{0}}-\frac{\tau_{b x}}{\rho_{0}}+h u_{s} \\
g \eta \frac{\partial d}{\partial y}-f u \bar{h}-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial y}-\frac{g h^{2}}{2 \rho_{0}} \frac{\partial \rho}{\partial y}-\frac{1}{\rho_{0}}\left(\frac{\partial s_{y x}}{\partial x}+\frac{\partial s_{y y}}{\partial y}\right) \\
+\frac{\tau_{s y}}{\rho_{0}}-\frac{\tau_{b y}}{\rho_{0}}+h v_{s}
\end{array}\right]
\end{align*}
$$

In Cartesian co-ordinates the system of 3D shallow water equations can be written

$$
\begin{equation*}
\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial \boldsymbol{F}_{x}^{I}}{\partial x^{\prime}}+\frac{\partial \boldsymbol{F}_{y}^{I}}{\partial y^{\prime}}+\frac{\partial \boldsymbol{F}_{\sigma}^{I}}{\partial \sigma}+\frac{\partial \boldsymbol{F}_{x}^{V}}{\partial x}+\frac{\partial \boldsymbol{F}_{y}^{V}}{\partial y}+\frac{\partial \boldsymbol{F}_{\sigma}^{V}}{\partial \sigma}=\boldsymbol{S} \tag{3.12}
\end{equation*}
$$

where the superscripts $I$ and $V$ denote the inviscid (convective) and viscous fluxes, respectively and where

$$
\begin{align*}
& \boldsymbol{U}=\left[\begin{array}{l}
h \\
h u \\
h v
\end{array}\right] \text {, } \\
& \boldsymbol{F}_{x}^{I}=\left[\begin{array}{l}
h \bar{u} \\
h u^{2}+\frac{1}{2} g\left(h^{2}-d^{2}\right) \\
h u v
\end{array}\right], \quad \boldsymbol{F}_{x}{ }^{V}=\left[\begin{array}{l}
0 \\
h A\left(2 \frac{\partial u}{\partial x}\right) \\
h A\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)
\end{array}\right] \\
& \boldsymbol{F}_{y}{ }^{I}=\left[\begin{array}{l}
h \bar{v} \\
h v u \\
h v^{2}+\frac{1}{2} g\left(h^{2}-d^{2}\right)
\end{array}\right], \quad \boldsymbol{F}_{y}{ }^{V}=\left[\begin{array}{l}
0 \\
h A\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \\
h A\left(2 \frac{\partial v}{\partial x}\right)
\end{array}\right]  \tag{3.13}\\
& \boldsymbol{F}_{\sigma}{ }^{I}=\left[\begin{array}{l}
h \omega \\
h \omega u \\
h \omega v
\end{array}\right], \quad \boldsymbol{F}_{\sigma}{ }^{v}=\left[\begin{array}{l}
0 \\
\frac{v_{t}}{h} \frac{\partial u}{\partial \sigma} \\
\frac{v_{t}}{h} \frac{\partial v}{\partial \sigma}
\end{array}\right] \\
& \boldsymbol{S}=\left[\begin{array}{l}
0 \\
g \eta \frac{\partial d}{\partial x}+f v h-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial x^{\prime}}-\frac{h g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial x} d z-\frac{1}{\rho_{0}}\left(\frac{\partial s_{x x}}{\partial x}+\frac{\partial s_{x y}}{\partial y}\right) \\
g \eta \frac{\partial d}{\partial y}-f u h-\frac{h}{\rho_{0}} \frac{\partial p_{a}}{\partial y^{\prime}}-\frac{h g}{\rho_{0}} \int_{z}^{\eta} \frac{\partial \rho}{\partial y} d z-\frac{1}{\rho_{0}}\left(\frac{\partial s_{y x}}{\partial x}+\frac{\partial s_{y y}}{\partial y},\right.
\end{array}\right.
\end{align*}
$$

Integrating Eq. (3.9) over the $i$ th cell and using Gauss's theorem to rewrite the flux integral gives

$$
\begin{equation*}
\int_{A_{i}} \frac{\partial \boldsymbol{U}}{\partial t} d \Omega+\int_{\Gamma_{i}}(\boldsymbol{F} \cdot \boldsymbol{n}) d s=\int_{A_{i}} \boldsymbol{S}(\boldsymbol{U}) d \Omega \tag{3.14}
\end{equation*}
$$

where $A_{i}$ is the area/volume of the cell $\Omega$ is the integration variable defined on $A_{i}, \Gamma_{i}$ is the boundary of the $i$ th cell and $d s$ is the integration variable along the boundary. $\boldsymbol{n}$ is the unit outward normal vector along the boundary. Evaluating the area/volume integrals by a one-point quadrature rule, the quadrature point being the centroid of the cell, and evaluating the boundary intergral using a mid-point quadrature rule, Eq. (3.14) can be written

$$
\begin{equation*}
\frac{\partial U_{i}}{\partial t}+\frac{1}{A_{i}} \sum_{j}^{N S} \boldsymbol{F} \cdot \boldsymbol{n} \Delta \Gamma_{j}=S_{i} \tag{3.15}
\end{equation*}
$$

Here $U_{i}$ and $S_{i}$, respectively, are average values of $U$ and $S$ over the $i$ th cell and stored at the cell centre, NS is the number of sides of the cell, $n_{j}$ is the unit outward normal vector at the $j$ th side and $\Delta \Gamma_{j}$ the length/area of the $j$ th interface.

Both a first order and a second order scheme can be applied for the spatial discretization.

For the 2D case an approximate Riemann solver (Roe's scheme, see Roe, 1981) is used to calculate the convective fluxes at the interface of the cells. Using the Roe's scheme the dependent variables to the left and to the right of an interface have to be estimated. Second-order spatial accuracy is achieved by employing a linear gradientreconstruction technique. The average gradients are estimated using the approach by Jawahar and Kamath, 2000. To avoid numerical oscillations a second order TVD slope limiter (Van Leer limiter, see Hirch, 1990 and Darwish, 2003) is used.

For the 3D case an approximate Riemann solver (Roe's scheme, see Roe, 1981) is used to calculate the convective fluxes at the vertical interface of the cells (x'y'-plane). Using the Roe's scheme the dependent variables to the left and to the right of an interface have to be estimated. Second-order spatial accuracy is achieved by employing a linear gradient-reconstruction technique. The average gradients are estimated using the approach by Jawahar and Kamath, 2000. To avoid numerical oscillations a second order TVD slope limiter (Van Leer limiter, see Hirch, 1990 and Darwish, 2003) is used. The convective fluxes at the horizontal interfaces (vertical line) are derived using first order upwinding for the low order scheme. For the higher order scheme the fluxes are approximated by the mean value of the fluxes calculated based on the cell values above and below the interface for the higher order scheme.

### 3.1.3 Transport equations

The transport equations arise in the salt and temperature model, the turbulence model and the generic transport model. They all share the form of Equation Eq. (2.20) in Cartesian coordinates. For the 2D case the integral form of the transport equation can be given by Eq. (3.9) where

$$
\begin{align*}
& \boldsymbol{U}=h \bar{C} \\
& \boldsymbol{F}^{I}=[h \bar{u} \bar{C}, \quad h \bar{\nu} \bar{C}] \\
& \boldsymbol{F}^{V}=\left[h D_{h} \frac{\partial \bar{C}}{\partial x}, \quad h D_{h} \frac{\partial \bar{C}}{\partial y}\right]  \tag{3.16}\\
& \boldsymbol{S}=-h k_{p} \bar{C}+h C_{s} S .
\end{align*}
$$

For the 3D case the integral form of the transport equation can be given by Eq. (3.9) where

$$
U=h C
$$

$$
\left.\begin{array}{l}
\boldsymbol{F}^{I}=\left[\begin{array}{lll}
h u C, & h v C, & h \omega C
\end{array}\right] \\
\boldsymbol{F}^{V}=\left[\begin{array}{ll}
h D_{h} \partial \frac{\partial C}{\partial x}, & h D_{h} \partial \frac{\partial C}{\partial y},
\end{array} \quad h \frac{D_{h}}{h} \partial \frac{\partial C}{\partial \sigma}\right. \tag{3.17}
\end{array}\right]
$$

The discrete finite volume form of the transport equation is given by Eq. (3.15). As for the shallow water equations both a first order and a second order scheme can be applied for the spatial discretization.

In 2D the low order approximation uses simple first order upwinding, i.e., element average values in the upwinding direction are used as values at the boundaries. The higher order version approximates gradients to obtain second order accurate values at the boundaries. Values in the upwinding direction are used. To provide stability and minimize oscillatory effects, a TVD-MUSCL limiter is applied (see Hirch, 1990, and Darwish, 2003).

In 3D the low order version uses simple first order upwinding. The higher order version approximates horizontal gradients to obtain second order accurate values at the horizontal boundaries. Values in the upwinding direction are used. To provide stability and minimize oscillatory effects, an ENO (Essentially Non-Oscillatory) type
procedure is applied to limit the horizontal gradients. In the vertical direction a $3^{\text {rd }}$ order ENO procedure is used to obtain the vertical face values (Shu, 1997).

### 3.2 Time Integration

Consider the general form of the equations

$$
\begin{equation*}
\frac{\partial \boldsymbol{U}}{\partial t}=\boldsymbol{G}(\boldsymbol{U}) \tag{3.18}
\end{equation*}
$$

For 2D simulations, there are two methods of time integration for both the shallow water equations and the transport equations: A low order method and a higher order method. The low order method is a first order explicit Euler method

$$
\begin{equation*}
\boldsymbol{U}_{n+1}=\boldsymbol{U}_{n}+\Delta t \boldsymbol{G}\left(\boldsymbol{U}_{n}\right) \tag{3.19}
\end{equation*}
$$

where $\Delta t$ is the time step interval. The higher order method uses a second order Runge Kutta method on the form:

$$
\begin{align*}
& \boldsymbol{U}_{n+\frac{1}{2}}=\boldsymbol{U}_{n}+\frac{1}{2} \Delta t \boldsymbol{G}\left(\boldsymbol{U}_{n}\right) \\
& \boldsymbol{U}_{n+1}=\boldsymbol{U}_{n}+\Delta t \boldsymbol{G}\left(\boldsymbol{U}_{n+\frac{1}{2}}\right) \tag{3.20}
\end{align*}
$$

For 3D simulations the time integration is semi-implicit. The horizontal terms are treated implicitly and the vertical terms are treated implicitly or partly explicitly and partly implicitly. Consider the equations in the general semi-implicit form.

$$
\begin{equation*}
\frac{\partial \boldsymbol{U}}{\partial t}=\boldsymbol{G}_{h}(\boldsymbol{U})+\boldsymbol{G}_{v}(B \boldsymbol{U})=\boldsymbol{G}_{h}(\boldsymbol{U})+\boldsymbol{G}_{v}^{I}(\boldsymbol{U})+\boldsymbol{G}_{v}^{V}(\boldsymbol{U}) \tag{3.21}
\end{equation*}
$$

where the $h$ and $v$ subscripts refer to horizontal and vertical terms, respectively, and the superscripts refer to invicid and viscous terms, respectively. As for 2D simulations, there is a lower order and a higher order time integration method.

The low order method used for the 3D shallow water equations can written as

$$
\begin{equation*}
\boldsymbol{U}_{n+1}-\frac{1}{2} \Delta t\left(\boldsymbol{G}_{v}\left(\boldsymbol{U}_{n+1}\right)+\boldsymbol{G}_{v}\left(\boldsymbol{U}_{n}\right)\right)=\boldsymbol{U}_{n}+\Delta t \boldsymbol{G}_{h}\left(\boldsymbol{U}_{n}\right) \tag{3.22}
\end{equation*}
$$

The horizontal terms are integrated using a first order explicit Euler method and the vertical terms using a second order implicit trapezoidal rule. The higher order method can be written

$$
\begin{align*}
& \boldsymbol{U}_{n+1 / 2}-\frac{1}{4} \Delta t\left(\boldsymbol{G}_{v}\left(\boldsymbol{U}_{n+1 / 2}\right)+\boldsymbol{G}_{v}\left(\boldsymbol{U}_{n}\right)\right)=\boldsymbol{U}_{n}+\frac{1}{2} \Delta t \boldsymbol{G}_{h}\left(\boldsymbol{U}_{n}\right)  \tag{3.23}\\
& \boldsymbol{U}_{n+1}-\frac{1}{2} \Delta t\left(\boldsymbol{G}_{v}\left(\boldsymbol{U}_{n+1}\right)+\boldsymbol{G}_{v}\left(\boldsymbol{U}_{n}\right)\right)=\boldsymbol{U}_{n}+\Delta t \boldsymbol{G}_{h}\left(\boldsymbol{U}_{n+1 / 2}\right)
\end{align*}
$$

The horizontal terms are integrated using a second order Runge Kutta method and the vertical terms using a second order implicit trapezoidal rule.

The low order method used for the 3D transport equation can written as

$$
\begin{equation*}
\boldsymbol{U}_{n+1}-\frac{1}{2} \Delta t\left(\boldsymbol{G}_{v}^{V}\left(\boldsymbol{U}_{n+1}\right)+\boldsymbol{G}_{v}^{V}\left(\boldsymbol{U}_{n}\right)\right)=\boldsymbol{U}_{n}+\Delta t \boldsymbol{G}_{h}\left(\boldsymbol{U}_{n}\right)+\Delta t \boldsymbol{G}_{v}^{I}( \tag{3.24}
\end{equation*}
$$

The horizontal terms and the vertical convective terms are integrated using a first order explicit Euler method and the vertical viscous terms are integrated using a second order implicit trapezoidal rule. The higher order method can be written

$$
\begin{align*}
& \boldsymbol{U}_{n+1 / 2}-\frac{1}{4} \Delta t\left(\boldsymbol{G}_{v}^{V}\left(\boldsymbol{U}_{n+1 / 2}\right)+\boldsymbol{G}_{v}^{V}\left(\boldsymbol{U}_{n}\right)\right)= \\
& \boldsymbol{U}_{n}+\frac{1}{2} \Delta t \boldsymbol{G}_{h}\left(\boldsymbol{U}_{n}\right)+\frac{1}{2} \Delta t \boldsymbol{G}_{v}^{I}\left(\boldsymbol{U}_{n}\right) \\
& \boldsymbol{U}_{n+1}-\frac{1}{2} \Delta t\left(\boldsymbol{G}_{v}^{V}\left(\boldsymbol{U}_{n+1}\right)+\boldsymbol{G}_{v}^{V}\left(\boldsymbol{U}_{n}\right)\right)=  \tag{3.25}\\
& \boldsymbol{U}_{n}+\Delta t \boldsymbol{G}_{h}\left(\boldsymbol{U}_{n+1 / 2}\right)+\Delta t \boldsymbol{G}_{v}^{I}\left(\boldsymbol{U}_{n+1 / 2}\right)
\end{align*}
$$

The horizontal terms and the vertical convective terms are integrated using a second order Runge Kutta method and the vertical terms are integrated using a second order implicit trapezoidal rule for the vertical terms.

### 3.3 Boundary Conditions

### 3.3.1 Closed boundaries

Along closed boundaries (land boundaries) normal fluxes are forced to zero for all variables. For the momentum equations this leads to fullslip along land boundaries.

### 3.3.2 Open boundaries

The open boundary conditions can be specified either in form of a unit discharge or as the surface elevation for the hydrodynamic equations. For transport equations either a specified value or a specified gradient can be given.

### 3.3.3 Flooding and drying

The approach for treatment of the moving boundaries problem (flooding and drying fronts) is based on the work by Zhao et al. (1994) and Sleigh et al. (1998). When the depths are small the problem is reformulated and only when the depths are very small the elements/cells are removed from the calculation. The reformulation is made by setting the momentum fluxes to zero and only taking the mass fluxes into consideration.

The depth in each element/cell is monitored and the elements are classified as dry, partially dry or wet. Also the element faces are monitored to identify flooded boundaries.

- An element face is defined as flooded if the following two criteria are satisfied: Firstly, the water depth at one side of face must be less than a tolerance depth, $h_{d r y}$, and the water depth at the other side of the face larger than a tolerance depth, $h_{\text {flood }}$. Secondly, the sum of the still water depth at the side for which the water depth is less than $h_{d r y}$ and the surface elevation at the other side must be larger than zero.
- An element is dry if the water depth is less than a tolerance depth, $h_{d r y}$, and no of the element faces are flooded boundaries. The element is removed from the calculation.
- An element is partially dry if the water depth is larger than $h_{d r y}$ and less than a tolerance depth, $h_{\text {wet }}$, or when the depth is less than the $h_{d r y}$ and one of the element faces is a flooded boundary. The momentum fluxes are set to zero and only the mass fluxes are calculated.
- An element is wet if the water depth is greater than $h_{\text {wet }}$. Both the mass fluxes and the momentum fluxes are calculated.

The wetting depth, $h_{\text {wet }}$, must be larger than the drying depth, $h_{d r y}$, and flooding depth, $h_{\text {flood }}$, must satisfy

$$
\begin{equation*}
h_{d y}<h_{\text {food }}<h_{w e t} \tag{3.26}
\end{equation*}
$$

The default values are $h_{d r y}=0.005 \mathrm{~m}, h_{\text {flood }}=0.05 \mathrm{~m}$ and $h_{\text {wet }}=0.1 \mathrm{~m}$.
Note, that for very small values of the tolerance depth, $h_{\text {wet }}$, unrealistically high flow velocities can occur in the simulation and give cause to stability problems.

4 VALIDATION

The new finite-volume model has been successfully tested in a number of basic, idealised situations for which computed results can be compared with analytical solutions or information from the literature. The model has also been applied and tested in more natural geophysical conditions; ocean scale, inner shelves, estuaries, lakes and overland, which are more realistic and complicated than academic and laboratory tests. A detailed validation report is under preparation.

This chapter presents a comparison between numerical model results and laboratory measurements for a dam-break flow in an L-shaped channel.

Additional information on model validation and applications can be found here
http://mikebydhi.com/Download/DocumentsAndTools/PapersAndDocs.aspx

### 4.1 Dam-break Flow through Sharp Bend

The physical model to be studied combines a square-shaped upstream reservoir and an L-shaped channel. The flow will be essentially twodimensional in the reservoir and at the angle between the two reaches of the L-shaped channel. However, there are numerical and experimental evidences that the flow will be mostly unidimensional in both rectilinear reaches. Two characteristics or the dam-break flow are of special interest, namely

- The "damping effect" of the corner
- The upstream-moving hydraulic jump which forms at the corner

The multiple reflections of the expansion wave in the reservoir will also offer an opportunity to test the 2D capabilities of the numerical models. As the flow in the reservoir will remain subcritical with relatively small-amplitude waves, computations could be checked for excessive numerical dissipation.

### 4.1.1 Physical experiments

A comprehensive experimental study of a dam-break flow in a channel with a 90 bend has been reported by Frazão and Zech (2002, 1999a, 1999b). The channel is made of a 3.92 and a 2.92 metre long and 0.495 metre wide rectilinear reaches connected at right angle by a $0.495 \times 0.495 \mathrm{~m}$ square element. The channel slope is equal to zero. A guillotine-type gate connects this L-shaped channel to a $2.44 \times 2.39 \mathrm{~m}$
(nearly) square reservoir. The reservoir bottom level is 33 cm lower that the channel bed level. At the downstream boundary a chute is placed. See the enclosed figure for details.

Frazão and Zech performed measurements for both dry bed and wet bed condition. Here comparisons are made for the case where the water in the reservoir is initially at rest, with the free surface 20 cm above the channel bed level, i.e. the water depth in the reservoir is 53 cm . The channel bed is initially dry. The Manning coefficients evaluated through steady-state flow experimentation are 0.0095 and $0.0195 \mathrm{~s} / \mathrm{m}^{1 / 3}$, respectively, for the bed and the walls of the channel.

The water level was measured at six gauging points. The locations of the gauges are shown in Figure 4.1 and the co-ordinates are listed in Table 4.1.


Figure 4.1 Set-up of the experiment by Frazão and Zech (2002)

Table 4.1 Location of the gauging points

| Location | $x(\mathrm{~m})$ | $y(\mathrm{~m})$ |
| :---: | :---: | :---: |
| T1 | 1.19 | 1.20 |
| T2 | 2.74 | 0.69 |
| T3 | 4.24 | 0.69 |
| T4 | 5.74 | 0.69 |
| T5 | 6.56 | 1.51 |
| T6 | 6.56 | 3.01 |

### 4.1.2 Numerical experiments

Simulations are performed using both the two-dimensional and the three-dimensional shallow water equations.

An unstructured mesh is used containing 18311 triangular elements and 9537 nodes. The minimum edge length is 0.01906 m and the maximum edge length is 0.06125 m . In the 3D simulation 10 layers is used for the vertical discretization. The time step is 0.002 s . At the downstream boundary, a free outfall (absorbing) boundary condition is applied. The wetting depth, flooding depth and drying depth are 0.002 $\mathrm{m}, 0.001 \mathrm{~m}$ and 0.0001 m , respectively.

A constant Manning coefficient of $105.26 \mathrm{~m}^{1 / 3} / \mathrm{s}$ is applied in the 2D simulations, while a constant roughness height of $5 \cdot 10^{08} \mathrm{~m}$ is applied in the 3D simulation.

### 4.1.3 Results

In Figure 4.2 time series of calculated surface elevations at the six gauges locations are compared to the measurements. In Figure 4.3 contour plots of the surface elevations are shown at $\mathrm{T}=1.6,3.2$ and 4.8 s (two-dimensional simulation).

In Figure 4.4 a vector plot and contour plots of the current speed at a vertical profile along the centre line (from ( $\mathrm{x}, \mathrm{y}$ ) $=(5.7,0.69$ ) to $(\mathrm{x}, \mathrm{y})=(6.4,0.69)$ ) at $\mathrm{T}=6.4 \mathrm{~s}$ is shown.


Figure 4.2 Time evolution of the water level at the six gauge locations. (blue) 3D calculation, (black) 2D calculation and (red) Measurements by Frazão and Zech (1999a,b)


Figure 4.3 Contour plots of the surface elevation at $T=1.6 \mathrm{~s}$ (top), $T=3.2 \mathrm{~s}$ (middle) and $T=4.8 \mathrm{~s}$ (bottom).


Figure 4.4 Vector plot and contour plots of the current speed at a vertical profile along the centre line at $T=6.4 \mathrm{~s}$

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## Basic Steps in the MIKE3-FM Temperature Modeling of Don Pedro Reservoir



Summary of data needed for Don Pedro Reservoir 3-D temperature model.

| Required Data | Source | In Project Database |
| :---: | :---: | :---: |
| Physical and Geomorphological |  |  |
| Bathymetry | Field survey | yes |
| Digital Terrain Model (IFSAR) | INTERMAP® | yes |
| Outlet (invert elevation) | Design drawings | yes |
| Outlet (lat/long) | Design drawings | yes |
| Dam spillway (elevation) | Design drawings | yes |
| Dam spillway (length, type) | Design drawings | yes |
| Old Don Pedro Dam spillway (elevation) | Design drawings or bathymetric survey | yes |
| Old Don Pedro Dam spillway (length, type) | Design drawings or bathymetric survey | yes |
| Old Don Pedro Dam crest (elevation) | Design drawings or bathymetric survey | yes |
| Old Don Pedro Dam crest (length, type) | Design drawings or bathymetric survey | yes |
| Old Don Pedro outlet (elevation) | TID | yes |
| Old Don Pedro outlet (lat/long) | USGS Topographical Map | yes |
| Flow and Operations |  |  |
| Tuolumne River upstream of reservoir (regulated) | CCSF, TID | yes |
| Tuolumne River upstream of reservoir (total) | TID | yes |
| Storage (daily) | USGS | yes |
| Withdrawals through powerhouse (daily) | TID | yes |
| Temperature |  |  |
| Tuolumne River upstream of reservoir | $\begin{aligned} & \hline \text { TID (starting October 2010); CCSF, } \\ & \text { CDFG } \end{aligned}$ | yes |
| Tributaries: Rough \& Ready, Moccasin, Sullivan and Woods Creeks | TID (starting April 2011) | yes |
| Profiles at several locations | CDFG, TID (starting May 2011) | yes |
| Meteorology |  |  |
| Air temperature, wind speed/direction, solar radiation, relative humidity | TID (starting November 2010); WRCC | yes |
| CCSF City and County of San Francisco <br> CDFG California Department of Fish an <br> TID Turlock Irrigation District <br> USGS U.S. Geological Survey <br> WRCC Western Regional Climate Center |  |  |


| From: | Staples, Rose |
| :---: | :---: |
| Sent: | Friday, April 06, 2012 6:48 PM |
| To: | 'Alves, Jim'; 'Anderson, Craig'; 'Asay, Lynette'; 'Aud, John'; 'Barnes, James'; 'Barnes, Peter'; 'Blake, Martin'; 'Bond, Jack'; Borovansky, Jenna; 'Boucher, Allison'; 'Bowes, Stephen'; 'Bowman, Art'; 'Brenneman, Beth'; 'Brewer, Doug'; 'Buckley, John'; 'Buckley, Mark'; 'Burt, Charles'; 'Byrd, Tim'; 'Cadagan, Jerry'; 'Carlin, Michael'; 'Charles, Cindy'; 'Cismowski, Gail'; 'Colvin, Tim'; 'Costa, Jan'; 'Cowan, Jeffrey'; 'Cox, Stanley Rob'; 'Cranston, Peggy'; 'Cremeen, Rebecca'; 'Day, Kevin'; 'Day, P'; 'Denean'; 'Derwin, Maryann Moise'; Devine, John; 'Donaldson, Milford Wayne'; 'Dowd, Maggie'; 'Drekmeier, Peter'; 'Edmondson, Steve'; 'Eicher, James'; 'Ferrari, Chandra'; 'Fety, Lauren'; 'Findley, Timothy'; 'Fuller, Reba'; 'Furman, Donn W'; 'Ganteinbein, Julie'; 'Giglio, Deborah'; 'Gorman, Elaine'; 'Grader, Zeke'; 'Gutierrez, Monica'; 'Hackamack, Robert'; 'Hastreiter, James'; 'Hatch, Jenny'; 'Hayat, Zahra'; 'Hayden, Ann'; 'Hellam, Anita'; 'Heyne, Tim'; 'Holley, Thomas'; 'Holm, Lisa'; 'Horn, Jeff'; 'Horn, Timi'; 'Hudelson, Bill'; 'Hughes, Noah'; 'Hughes, Robert'; 'Hume, Noah'; 'Jackman, Jerry'; 'Jackson, Zac'; 'Jennings, William'; 'Jensen, Art'; 'Jensen, Laura'; 'Johannis, Mary'; 'Johnson, Brian'; 'Justin'; 'Keating, Janice'; 'Kempton, Kathryn'; 'Kinney, Teresa'; 'Koepele, Patrick'; 'Kordella, Lesley'; 'Lein, Joseph'; 'Levin, Ellen'; 'Lewis, Reggie'; 'Linkard, David'; 'Looker, Mark'; 'Lwenya, Roselynn'; 'Lyons, Bill'; 'Madden, Dan'; 'Manji, Annie'; 'Marko, Paul'; 'Marshall, Mike'; 'Martin, Michael'; 'Martin, Ramon'; 'Mathiesen, Lloyd'; 'McDaniel, Dan'; 'McDevitt, Ray'; 'McDonnell, Marty'; 'McLain, Jeffrey'; 'Means, Julie'; 'Mills, John'; 'Morningstar Pope, Rhonda'; 'Motola, Mary'; 'O'Brien, Jennifer'; 'Orvis, Tom'; 'Ott, Bob'; 'Ott, Chris'; 'Paul, Duane'; 'Pavich, Steve'; 'Pinhey, Nick'; 'Pool, Richard'; 'Porter, Ruth'; 'Powell, Melissa'; 'Puccini, Stephen'; 'Raeder, Jessie'; 'Ramirez, Tim'; 'Rea, Maria'; 'Reed, Rhonda'; 'Richardson, Kevin'; 'Ridenour, Jim'; 'Robbins, Royal'; 'Romano, David O'; 'Roos-Collins, Richard'; 'Roseman, Jesse'; 'Rothert, Steve'; 'Sandkulla, Nicole'; 'Saunders, Jenan'; 'Schutte, Allison'; 'Sears, William'; 'Shakal, Sarah'; 'Shipley, Robert'; 'Shumway, Vern'; 'Shutes, Chris'; 'Sill, Todd'; 'Slay, Ron'; 'Smith, Jim'; Staples, Rose; 'Steindorf, Dave'; 'Steiner, Dan'; 'Stone, Vicki'; 'Stork, Ron'; 'Stratton, Susan'; 'Taylor, Mary Jane'; 'Terpstra, Thomas'; 'TeVelde, George'; 'Thompson, Larry'; 'Vasquez, Sandy'; 'Verkuil, Colette'; 'Vierra, Chris'; 'Walters, Eric'; 'Wantuck, Richard'; 'Welch, Steve'; 'Wesselman, Eric'; 'Wheeler, Dan'; 'Wheeler, Dave'; 'Wheeler, Douglas'; 'Wilcox, Scott'; 'Williamson, Harry'; 'Willy, Allison'; 'Wilson, Bryan'; 'Winchell, Frank'; 'Wooster, John'; 'Workman, Michelle'; 'Yoshiyama, Ron'; 'Zipser, Wayne' |
| Subject: | LIVE MEETING LINK IF NOT ATTENDING DON PEDRO WORKSHOPS / MEETINGS IN PERSON |

If you are unable to attend the upcoming Don Pedro workshops and meetings in person, you will be able to call in (see "AUDIO INFORMATION" below) and you can also use your computer to LIVE MEETING (see individual workshop/meeting links below). If you have not used LIVE MEETING before, please be sure, well before the meeting time, to click on "FIRST ONLINE MEETING" link below. Thank you.

## AUDIO INFORMATION

Use call-in number: 866-994-6437, Conference Code 5424697994

## ON-LINE MEETING INFORMATION

(On-Line Meetings will be open approximately half hour prior to the meeting start time to allow for any technical issues to be resolved. If you have not used On-Line Meeting in the past, please allow a few extra minutes for your first log-on.)

## Monday, April 9 (10 am - 5 pm): Hydrology Workshop (W\&AR-2)

```
Join online meeting
https://meet.hdrinc.com/jenna.borovansky/DG5FH7C1

\title{
Tuesday, April 10 (8:30-10:15am): Reservoir Temperature Modeling Data and Methods Consultation Meeting (W\&AR-3)
}
Join online meeting
https://meet.hdrinc.com/jenna.borovansky/QC5C5HN1
First online meeting?
Tuesday, April 10 (10:30am - 5:00pm) Salmonid Information Synthesis Workshop (W\&AR-5)
Join online meeting
https://meet.hdrinc.com/jenna.borovansky/37NNBCDP
First online meeting?
Wednesday, April 11 (9:00 am - Noon): Temperature Criteria Evaluation Consultation Meeting (W\&AR-14)
Join online meeting
https://meet.hdrinc.com/jenna.borovansky/MHVKDJYZ
First online meeting?

ROSE STAPLES
CAP-OM

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\begin{tabular}{|c|c|}
\hline From: & Staples, Rose \\
\hline Sent: & Thursday, July 26, 2012 4:39 PM \\
\hline \multirow[t]{33}{*}{To:} & Alves, Jim; Anderson, Craig; Asay, Lynette; Aud, John; Barnes, James; Barnes, Peter; \\
\hline & Beniamine Beronia; Blake, Martin; Bond, Jack; Borovansky, Jenna; Boucher, Allison; \\
\hline & Bowes, Stephen; Bowman, Art; Brenneman, Beth; Brewer, Doug; Buckley, John; \\
\hline & Buckley, Mark; Burt, Charles; Byrd, Tim; Cadagan, Jerry; Carlin, Michael; Charles, \\
\hline & Cindy; Colvin, Tim; Costa, Jan; Cowan, Jeffrey; Cox, Stanley Rob; Cranston, Peggy; \\
\hline & Cremeen, Rebecca; Damin Nicole; Day, Kevin; Day, P; Denean; Derwin, Maryann \\
\hline & Moise; Devine, John; Donaldson, Milford Wayne; Dowd, Maggie; Drekmeier, Peter; \\
\hline & Edmondson, Steve; Eicher, James; Fargo, James; Ferranti, Annee; Ferrari, Chandra; \\
\hline & Fety, Lauren; Findley, Timothy; Fuller, Reba; Furman, Donn W; Ganteinbein, Julie; \\
\hline & Giglio, Deborah; Gorman, Elaine; Grader, Zeke; Gutierrez, Monica; Hackamack, \\
\hline & Robert; Hastreiter, James; Hatch, Jenny; Hayat, Zahra; Hayden, Ann; Hellam, Anita; \\
\hline & Heyne, Tim; Holley, Thomas; Holm, Lisa; Horn, Jeff; Horn, Timi; Hudelson, Bill; \\
\hline & Hughes, Noah; Hughes, Robert; Hume, Noah; Jackman, Jerry; Jackson, Zac; Jennings, \\
\hline & William; Jensen, Art; Jensen, Laura; Johannis, Mary; Johnson, Brian; Justin; Keating, \\
\hline & Janice; Kempton, Kathryn; Kinney, Teresa; Koepele, Patrick; Kordella, Lesley; Lara, \\
\hline & Marco; Lein, Joseph; Levin, Ellen; Lewis, Reggie; Linkard, David; Looker, Mark; Loy, \\
\hline & Carin; Lwenya, Roselynn; Lyons, Bill; Madden, Dan; Manji, Annie; Marko, Paul; \\
\hline & Marshall, Mike; Martin, Michael; Martin, Ramon; Mathiesen, Lloyd; McDaniel, Dan; \\
\hline & McDevitt, Ray; McDonnell, Marty; McLain, Jeffrey; Mein Janis; Mills, John; Minami \\
\hline & Amber; Monheit, Susan; Morningstar Pope, Rhonda; Motola, Mary; Murphey, \\
\hline & Gretchen; O'Brien, Jennifer; Orvis, Tom; Ott, Bob; Ott, Chris; Paul, Duane; Pavich, \\
\hline & Steve; Pinhey, Nick; Pool, Richard; Porter, Ruth; Powell, Melissa; Puccini, Stephen; \\
\hline & Raeder, Jessie; Ramirez, Tim; Rea, Maria; Reed, Rhonda; Richardson, Kevin; Ridenour \\
\hline & Jim; Robbins, Royal; Romano, David O; Roos-Collins, Richard; Roseman, Jesse; \\
\hline & Rothert, Steve; Sandkulla, Nicole; Saunders, Jenan; Schutte, Allison; Sears, William; \\
\hline & Shakal, Sarah; Shipley, Robert; Shumway, Vern; Shutes, Chris; Sill, Todd; Slay, Ron; \\
\hline & Smith, Jim; Staples, Rose; Steindorf, Dave; Steiner, Dan; Stone, Vicki; Stork, Ron; \\
\hline & Stratton, Susan; Taylor, Mary Jane; Terpstra, Thomas; TeVelde, George; Thompson, \\
\hline & Larry; Vasquez, Sandy; Verkuil, Colette; Vierra, Chris; Wantuck, Richard; Welch, Steve; \\
\hline & Larry; Vasquez, Sandy; Verkuil, Colette; Vierra, Chris; Wantuck, Richard; Welch, Steve; \\
\hline & Wesselman, Eric; Wheeler, Dan; Wheeler, Dave; Wheeler, Douglas; Wilcox, Scott; \\
\hline & Williamson, Harry; Willy, Allison; Wilson, Bryan; Winchell, Frank; Wooster, John; \\
\hline & Workman, Michelle; Yoshiyama, Ron; Zipser, Wayne \\
\hline Subject: & Don Pedro Reservoir Temperature Model April 102012 Meeting Notes \\
\hline Attachments: & April 102012 Reserv Temp RP Mtg_120726r.pdf \\
\hline
\end{tabular}

Attached please find Meeting Notes from the Don Pedro Project Relicensing W\&AR-03 Reservoir Temperature Model meeting held on April 10, 2012.

Action items for the Districts that came out of the meeting are addressed within the notes and model development is proceeding. As requested by the Relicensing Participants, the Districts have postponed the next meeting (originally scheduled for September 18) until mid-October (now scheduled for October 26, 2012) when the Districts will present model verification/calibration, as well as conduct training in use of the model.

NOTE: A copy of this announcement, and the accompanying attachment, are also being uploaded to the INTRODUCTION/ANNOUNCEMENT section of the relicensing website www.donpedro-relicensing.com.
\begin{tabular}{|llll}
\hline To: & Don Pedro Relicensing Participants & & \\
\hline From: & \begin{tabular}{l} 
Turlock Irrigation District / Modesto Irrigation \\
District
\end{tabular} & Project: & Don Pedro Hydroelectric Project \\
\hline Date: & July 26, 2012 & & \\
\hline
\end{tabular}

\section*{Re: Don Pedro Project Reservoir Temperature Model (W\&AR-3) Meeting Summary. April 10, 2012}

\section*{Attendees:}

Scott Lowe, HDR
Guy Apicella, HDR
Dale Stanton, CDFG
Bill Johnston, MID
Art Goodwin
Steve Boyd, TID
Bob Nees, TID

\author{
Greg Dias, MID \\ Mike Maher, SWRCB \\ Tim O'Laughlin (on \\ phone) \\ Carin Loy, HDR \\ John Devine, HDR \\ Jenna Borovansky, HDR
}

On April 10, 2012, as identified in the FERC-approved study plan, the Districts hosted a meeting at Modesto Irrigation District's offices in Modesto, California, to provide an overview of the MIKE3 model being developed for the Don Pedro Relicensing Study W\&AR-3 Reservoir Temperature Modeling Study. Guy Apicella and Scott Lowe of HDR conducted the presentation in which the underlying model physics and assumptions were discussed, as well as the empirical data being used as input, and the nature of the output. Output from the model will consist of simulated temperature responses to changes in hydrological and meteorological drivers as well as simulated project operations.

The modeling task consists of five steps: (1) set up a 3-D flexible mesh; (2) compile input data; (3) calibrate the model; (4) verify the model; and (5) simulate model scenarios. At this time, HDR is setting up the 3-D flexible mesh and compiling all of the input data. Dr. Lowe showed examples of how the mesh looks, discussed some of the choices modelers have when building the mesh, and the two kinds of mesh layering that can be employed to build the vertical dimension of the model-sigma and sigma/z-layer. Using preliminary temperature and bathymetry data, Dr. Lowe also gave an example of how a temperature cross-section might look along an arbitrary line (not the thalweg), pointing out the different temperatures with depth. The differences in interpreting a graphical display of a cross section taken along the thalweg and a cross section taken arbitrarily, like in the example, were pointed out.

Three action items were identified during the meeting:
(1) The Districts will post the PowerPoint presentation from the meeting on the Project website. (Posted April 13, 2012, www.donpedro-relicensing.com)
(2) CDFG asked the Districts to provide examples of reservoirs where MIKE3 has been applied by HDR or others (described below).
(3) CDFG asked the Districts to contact DHI and ask if they know of projects where the model's predictions were compared to post-implementation monitoring data (described below).

\section*{Examples of MIKE3 Application in Reservoirs}

There are numerous examples of other reservoirs, lakes, impoundments, estuaries and bays where MIKE3 has been used for temperature modeling by HDR or others. A brief list of examples of where MIKE3 models have been used by others follows:
o Lake Ontario - various applications by the Ontario Ministry of Environment (OME) and Ontario Power Generation (OPG)
o Lake Simcoe (Ontario, Canada) - by W. F. Baird \& Associates
o Lake Michigan - by W. F. Baird \& Associates
o Lake Erie - by Great Lakes Unit of Ontario Ministry of Environment
o Hamilton Harbour (off Lake Ontario) - by Cheng He of National Water Research Institute (Canada)
o Lake Esrum (Denmark) - by Rikke Margrethe Closter, Ph.D. thesis at Danish Meteorological Institute
o Fehmarnbelt/Roskilde Fjord (Main entrance to Baltic Sea between Germany and Denmark) - by DHI
o Upper Adriatic Sea and Venice Lagoon (Italy) - by DHI
All the projects are examples of where the MIKE3 model was calibrated by comparing calculated temperature results to measured depth profiles. A description or full report for each of these projects is available by request.

HDR has used MIKE3 models for various projects. Recent applications of the MIKE3-FM model for temperature and/or flow velocities by the team developing the Don Pedro model are:
o Lake Champlain (Vermont and New York)
o Northfield Reservoir (Massachusetts)
o East River (New York) Power Plant Thermal Discharge
o Stalley Bay (St Thomas, USVI)
o Black Dog Cooling Lake (MN)
o Goodman Bay (Bahamas)
The Lake Champlain model was calibrated using measured temperature profiles; the Northfield Reservoir was not calibrated. HDR's report on the Lake Champlain model is posted on www.donpedro-relicensing.com. The Northfield Reservoir model write-up, which covered sediment transport, can also be found at www.donpedro-relicensing.com.

In addition to the temperature applications above, the HDR team has also applied MIKE3 to these waterways for non-temperature (e.g., water quality, sediment) modeling:
o Lake Jessup (FL)
o Jamaica Bay (NY)
o New York Harbor (NY)
o Long Island Sound (NY-CT-RI)
o Hudson River (NY)
o Newark Bay (NJ)

\section*{Model Predictions versus Actual}

CDFG also asked if the MIKE3 model temperature projections were ever compared to temperature monitoring data collected in the future (versus historical data). HDR could not find information that documented such a comparison and, when contacted, DHI was not able to provide examples.

\section*{Path Forward}

HDR indicated that model development was proceeding. The Districts will provide an update in the summer and then hold a meeting in mid-October to present model verification/calibration and conduct training in use of the model.```


[^0]:    1 Including the MIKE 21 Flow Model FM (two-dimensional flow) and MIKE 3 Flow Model FM (three-dimensional flow)

